2

NUMBER AND ALGEBRA

WORKING WITH NUMBERS

Numbers are interesting! You already know about odd and even numbers, prime and composite numbers and integers. There are also triangular and square numbers, Fibonacci numbers, perfect and amicable numbers, and complex numbers. Numbers can form special patterns such as in magic squares, Pythagorean triads and Pascal's triangle. These patterns can be found in biology, probability, digital technology and software design.

In this chapter, we will revise and extend our number skills and examine the patterns involved when calculating with powers, using mental calculation, pen-and-paper methods and calculators.

Chapter outline

	Working mathematically				
2.01 Mental calculation	U	F	PS	R	
2.02 Adding and subtracting integers	U	F	PS	R	
2.03 Multiplying and dividing integers	U	F		R	C
2.04 Order of operations	U	F	PS	R	C
2.05 Decimals	U	F	PS	R	
2.06 Multiplying and dividing decimals	U	F	PS	R	
2.07 Terminating and recurring decimals	U	F		R	C
2.08 Powers and roots	U	F		R	C
2.09 Factor trees	U	F		R	C
2.10 Index laws for multiplying and dividing	U	F		R	C
2.11 More index laws	U	F		R	C

Wordbank

base (in index notation) A number raised to a power, for example, in 3^5, 3 is the base

cube root The value which, if cubed, will give the number required, for example $\sqrt[3]{8} = 2$ because $2^3 = 8$

factor tree A diagram that lists the prime factors of a number.

index notation A way of writing powers for the repeated multiplication of a number, for example, 3^5.

mental calculation To operate with numbers 'in your head', without using a pen or calculator

order of operations The rules for calculating an expression involving mixed operations, such as $14 - 2 \times 4 + 1$

recurring decimal A decimal that has one or more digits that repeat endlessly

terminating decimal A decimal that is not recurring, but comes to an end

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

- apply number laws to help mental calculation
- calculate with integers, decimals and fractions, using mental and written strategies and technology
- apply the order of operations to evaluate mixed expressions
- investigate terminating and recurring decimals
- evaluate expressions involving powers, square root and cube root, after first estimating
- find square roots and cube roots of any non-square whole number using a calculator, after first estimating
- explore the properties of squares and square roots of products: $(ab)^2$ and $\sqrt{ab}$
- use a factor tree to write a number as a product of its prime factors
- use factor trees to find the highest common factor and lowest common multiple of 2 or more numbers, and to find the square root or cube root of a square number or cube number respectively
- explore the index laws with positive and zero powers

SkillCheck ANSWERS ON P. 545

Decimals

Fractions and Decimals

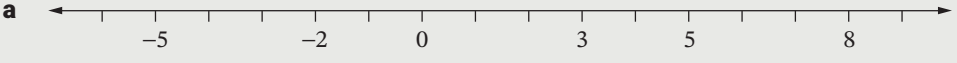
Calculation aids

Number grids

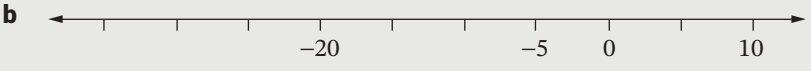
Magic squares

1 Copy and complete each number line.

a



b



2 Write these integers in ascending order: −7, 6, −5, 11, −2, 1, 0, 7.

3 Write these integers in descending order: 12, −3, 5, −1, 2, −9, 4, 8.

4 In the decimal 2.718, name the digit in the:

a tenths place **b** thousandths place **c** hundredths place

5 Convert each decimal to a fraction.

a 0.03 **b** 0.007 **c** 0.49 **d** 0.9

6 How many decimal places has each number?

a 14.451 **b** 86.6 **c** 28.04 **d** 3.141 59

7 **a** List the factors of 15.

b List the first 5 multiples of 15.

8 State whether each number is prime or composite.

a 2 **b** 17 **c** 25 **d** 27

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Mental calculation 2.01

The word 'mental' means 'using the mind', and mental calculation is the skill of working with numbers 'in your head', without using a pen or calculator. In the 'Mental skills' sections of *New Century Maths 7*, you learnt many mental strategies for calculating with numbers. Let's revise them now.

Example 1 Estimating answers by rounding numbers

a $15 + 37 + 18 + 45 + 22 \approx 20 + 40 + 20 + 40 + 20$
$= 140$ (exact answer = 137)

b $18 \times 12 \approx 20 \times 11$
$= 220$ (exact answer = 216)

c $504 \div 8 \approx 500 \div 10$
$= 50$ (exact answer = 63)

Example 2 Adding and multiplying numbers in any order

The commutative laws: $a + b = b + a$ and $a \times b = b \times a$

The associative laws: $(a + b) + c = a + (b + c)$ and $(a \times b) \times c = a \times (b \times c)$

a $15 + 37 + 18 + 45 + 22 = (15 + 45) + (18 + 22) + 37$
$= 60 + 40 + 37$
$= 137$
Pair numbers that add to multiples of 10.

b $7 \times 4 \times 5 = 7 \times (4 \times 5)$
$= 7 \times 20$
$= 140$
Pair numbers that multiply to multiples of 10.

Example 3 Adding and subtracting 8 or 9

a $43 + 9 = 43 + 10 - 1$ Add 10, count back 1.
$= 53 - 1$
$= 52$

b $97 + 8 = 97 + 10 - 2$ Add 10, count back 2.
$= 107 - 2$
$= 105$

c $61 - 8 = 61 - 10 + 2$ Subtract 10, count forward 2.

$= 51 + 2$

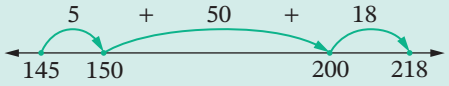
$= 53$

Example 4 Calculating differences using a number line

$218 - 145$

$145 + \_\_\_ = 218?$

Think of this as the difference or 'gap' between 145 and 218 on a number line.



Make smaller jumps.

Add the jumps.

$5 + 50 + 18 = 73$

This is like counting out change with money.

So $218 - 145 = 73$

Example 5 Multiplying and dividing by a multiple of 10

a $9 \times 60 = 9 \times 6 \times 10$

$= 54 \times 10$

$= 540$

b $2700 \div 30 = 2700 \div 30$

$= 270 \div 3$

$= 90$

Example 6 Doubling and halving numbers, Multiplying and dividing by 4

a $47 \times 2 = 40 \times 2 + 7 \times 2$

$= 80 + 14$

$= 94$

b $\frac{1}{2} \times 68 = \frac{1}{2} \times 60 + \frac{1}{2} \times 8$

$= 30 + 4$

$= 34$

c $\frac{1}{2} \times 192 = \frac{1}{2} \times 180 + \frac{1}{2} \times 12$

$= 90 + 6$

$= 96$

d $29 \times 4 = 29 \times 2 \times 2$ Double twice

Double 29 = 58, double 58 = 116

$29 \times 4 = 116$

e $16 \times 8 = 16 \times 2 \times 2 \times 2$ Double 3 times

Double 16 = 32, double 32 = 64, double 64 = 128

$16 \times 8 = 128$

f $560 \div 4 = 560 \div 2 \div 2$ Halve twice

$\frac{1}{2} \times 560 = 280$, $\frac{1}{2} \times 280 = 140$

$560 \div 4 = 140$

Example 7 Simplifying multiplication by factorising

a $45 \times 16 = 9 \times 5 \times 4 \times 4$
$= 5 \times 4 \times 4 \times 9$
$= 20 \times 36$
$= 720$

b $12 \times 18 = 6 \times 2 \times 9 \times 2$
$= 6 \times 9 \times 2 \times 2$
$= 54 \times 4$
$= 216$

Example 8 Multiplying and dividing by 5, 15, 25, 50

a $16 \times 15 = 16 \times \frac{1}{2} \times 30$ or $8 \times 2 \times 15$
$= 8 \times 30$
$= 240$

b $24 \times 25 = 24 \times \frac{1}{4} \times 100$ or $6 \times 4 \times 25$
$= 6 \times 100$
$= 600$

c $420 \div 5 = 420 \div 10 \times 2$
$= 42 \times 2$
$= 84$

d $300 \div 25 = 300 \div 100 \times 4$
$= 3 \times 4$
$= 12$

Example 9 Multiplying by 8, 9, 11, 12

The distributive laws:

$$a \times (b + c) = a \times b + a \times c \quad \text{and} \quad a \times (b - c) = a \times b - a \times c$$

a $25 \times 9 = 25 \times (10 - 1)$
$= 25 \times 10 - 25 \times 1$
$= 250 - 25$
$= 225$

b $18 \times 8 = 18 \times (10 - 2)$
$= 18 \times 10 - 18 \times 2$
$= 180 - 36$
$= 144$

c $13 \times 11 = 13 \times (10 + 1)$
$= 13 \times 10 + 13 \times 1$
$= 130 + 13$
$= 143$

d $27 \times 12 = 27 \times (10 + 2)$
$= 27 \times 10 + 27 \times 2$
$= 270 + 54$
$= 324$

Short and long division

Short division is a technique for dividing by one-digit numbers (whole numbers less than 10).

Long division is a technique for dividing by numbers with 2 or more digits (whole numbers greater than 10).

Example 10

Evaluate each quotient.

a $520 \div 8$ **b** $945 \div 21$

Solution

a By short division:

$8\overline{)520}$ — 8 into 5 goes 0

$8\overline{)52^{4}0}$ (6) — 8 into 52 goes 6, remainder 4

$8\overline{)52^{4}0}$ (6 5) — 8 into 40 goes 5 (exactly)

$520 \div 8 = 65$ — Check by estimating: $520 \div 8 \approx 500 \div 10 = 50$ (65 is close to 50)

OR:

$520 \div 8 = 520 \div 2 \div 2 \div 2$ — Halve three times

$520 \div 2 = 260$, $260 \div 2 = 130$, $130 \div 2 = 65$

$520 \div 8 = 65$

b By long division:

$$\begin{array}{r} 45 \\ 21\overline{)\,945} \\ -84\downarrow \\ \hline 105 \\ -105 \\ \hline 0 \end{array}$$

21 into 94 is 4, remainder 10.
21 into 105 is 5.

$945 \div 21 = 45$ — Check by estimating: $945 \div 21 \approx 900 \div 20 = 45$

OR:

Guessing with 'easy' multiples of 21.

$21\overline{)\,945}$	
-420	20 times
525	
-420	20 times
105	
-84	4 times
21	
-21	1 time
0	45 times

EXERCISE 2.01 ANSWERS ON P. 545

Mental calculation U F PS R

1 Estimate the value of each expression, then check your estimate with a calculator. EXAMPLE 1

a $27 + 11 + 87 + 142 + 64$ **b** $55 + 34 - 22 - 46 + 136$

c $684 + 903$ **d** $35 + 81 + 110 + 22 + 7$

e $517 - 96$ **f** $766 - 353$

g 46×8 **h** 367×2

i 51×12 **j** $245 \div 7$

k $828 \div 3$ **l** $564 \div 12$

2 Evaluate each sum. PS R EXAMPLE 2

a $55 + 18 + 25 + 9 + 12$ **b** $23 + 42 + 16 + 24 + 7$

c $140 + 33 + 12 + 20 + 28$ **d** $37 + 11 + 29 + 5 + 13$

e $45 + 49 + 121 + 25 + 10$ **f** $74 + 99 + 21 + 32 + 16$

3 What is the value of $720 + 1456$? Select the correct answer **A**, **B**, **C** or **D**.

A 2176 **B** 2206 **C** 2806 **D** 8656

4 Evaluate each expression. R EXAMPLE 3

a $27 + 9$	**b** $45 + 9$	**c** $16 + 8$	**d** $34 + 28$
e $17 + 11$	**f** $22 + 21$	**g** $69 + 12$	**h** $114 + 32$
i $45 - 9$	**j** $27 - 9$	**k** $76 - 8$	**l** $32 - 18$
m $55 - 12$	**n** $183 - 11$	**o** $67 - 41$	**p** $121 - 22$

5 Evaluate each difference. PS R EXAMPLE 4

a $560 - 327$ **b** $1614 - 239$ **c** $452 - 367$

6 Write the answer to each multiplication.

a 3×8	**b** 4×4	**c** 7×5	**d** 6×9
e 4×9	**f** 8×5	**g** 8×8	**h** 7×3
i 4×6	**j** 6×6	**k** 9×3	**l** 7×4

7 Write the answer to each division.

a $49 \div 7$	**b** $20 \div 4$	**c** $36 \div 9$	**d** $27 \div 3$
e $54 \div 6$	**f** $28 \div 7$	**g** $15 \div 5$	**h** $72 \div 8$
i $16 \div 4$	**j** $45 \div 5$	**k** $24 \div 8$	**l** $40 \div 8$

8 Evaluate each product. PS R

a $8 \times 2 \times 50$	**b** $6 \times 5 \times 3$	**c** $25 \times 7 \times 4$	**d** $4 \times 8 \times 5$
e $5 \times 2 \times 10$	**f** $10 \times 4 \times 25$	**g** $15 \times 50 \times 2$	**h** $3 \times 4 \times 20$
i $6 \times 6 \times 5$	**j** $2 \times 5 \times 32$	**k** $20 \times 17 \times 5$	**l** $10 \times 7 \times 20$

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EXAMPLE 5

9 Evaluate each expression. R

a	8×20	**b**	7×500	**c**	2×9000	**d**	15×20
e	3×70	**f**	6×300	**g**	9×4000	**h**	12×200
i	$900 \div 30$	**j**	$1200 \div 200$	**k**	$250 \div 50$	**l**	$320 \div 40$
m	$1800 \div 90$	**n**	$2000 \div 20$	**o**	$42\,000 \div 6000$	**p**	$240 \div 80$

10 How many hours are there in one week? Select **A**, **B**, **C** or **D**.

A	84	**B**	151	**C**	168	**D**	240

EXAMPLE 6

11 Evaluate each expression. R

a	85×2	**b**	39×2	**c**	57×4	**d**	28×4
e	16×8	**f**	33×8	**g**	$\frac{1}{2} \times 648$	**h**	$\frac{1}{2} \times 232$
i	$216 \div 4$	**j**	$488 \div 4$	**k**	$872 \div 8$	**l**	$208 \div 8$

EXAMPLE 7

12 Evaluate each product by factorising. PS R

a	7×40	**b**	25×16	**c**	22×6	**d**	13×20
e	35×6	**f**	21×3	**g**	16×12	**h**	18×18

EXAMPLE 8

13 Evaluate each expression. PS R

a	22×5	**b**	14×5	**c**	28×50	**d**	36×15
e	44×25	**f**	52×50	**g**	$800 \div 50$	**h**	$90 \div 5$
i	$120 \div 15$	**j**	$1000 \div 25$	**k**	$410 \div 5$	**l**	$700 \div 50$

14 Nathan can type 76 words per minute. How many words can he type in 15 minutes?

EXAMPLE 9

15 Evaluate each product by multiplying by 10 first, then adding or subtracting. R

a	34×9	**b**	51×9	**c**	27×8	**d**	72×8
e	36×11	**f**	41×11	**g**	64×12	**h**	22×12

EXAMPLE 10

16 Evaluate each quotient.

a	$2640 \div 8$	**b**	$1638 \div 9$	**c**	$616 \div 7$	**d**	$1584 \div 4$
e	$675 \div 15$	**f**	$756 \div 14$	**g**	$806 \div 31$	**h**	$729 \div 27$

17 There are 135 students in Year 8. If they are divided evenly into 5 classes, how many students will be in each class? Select **A**, **B**, **C** or **D**.

A	21	**B**	25	**C**	27	**D**	29

18 Divide a restaurant bill of $204 evenly among 6 people.

Did you know?

Fibonacci numbers

Fibonacci numbers are a number pattern 1, 1, 2, 3, 5, 8, ... named after the Italian mathematician Fibonnacci (pronounced 'fibbon-archie'), also known as Leonardo of Pisa. He introduced this number pattern to Europe in 1202, although it was known earlier in India.

The Fibonacci sequence of numbers is calculated as follows:

Start with 1 and 1

$1 + 1 = 2$	1, 1, 2
$1 + 2 = 3$	1, 1, 2, 3
$2 + 3 = 5$	1, 1, 2, 3, 5
$3 + 5 = 8$	1, 1, 2, 3, 5, 8

Each number is the sum of the previous 2 numbers.

Surprisingly, Fibonacci numbers have proved useful in computer algorithms, the growth of plants, software development and optics, even though they were first calculated hundreds of years ago.

Calculate the next 5 Fibonacci numbers.

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19th century statue of Fibonacci today in Pisa, Italy.

Adding and subtracting integers 2.02

Integers are all the positive and negative whole numbers and zero. Integers can be added and subtracted on a number line. A negative number can be entered into a calculator using the (−) or +/− key.

Adding and subtracting integers

Adding a negative number is the same as subtracting its opposite: $7 + (-4) = 7 - 4 = 3$

Subtracting a negative number is the same as adding its opposite: $7 - (-4) = 7 + 4 = 11$

Integers

Integers using diagrams

Integers using coloured squares

Number lines

Adding directed numbers

Subtracting directed numbers

WS A page of number lines

Integer Snap

Adding integers

Subtracting integers

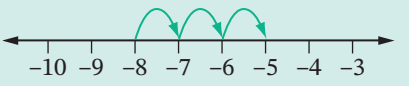
Example 11

Evaluate each expression.

a $-8 + 3$ **b** $-8 - 3$ **c** $2 + (-6)$ **d** $2 - (-6)$

Solution

a $-8 + 3 = -5$

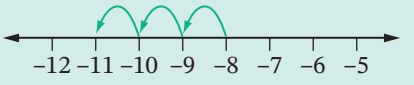


Start at −8, go forward 3

On a calculator, enter:

(–) 8 + 3 =.

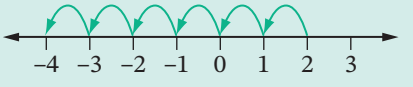
b $-8 - 3 = -11$



Start at −8, go back 3.

(–) 8 – 3 =.

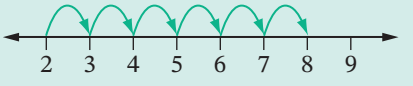
c $2 + (-6) = 2 - 6 = -4$



Start at 2, go back 6.

2 + (–) 6 =.

d $2 - (-6) = 2 + 6 = 8$



Start at 2, go forward 6.

2 – (–) 6 =.

EXERCISE 2.02 ANSWERS ON P. 546

Adding and subtracting integers U F PS R

EXAMPLE 11

1 Evaluate each sum and check your answer using a calculator.

A **sum** is the answer to an addition

a $-3 + 1$	**b** $-5 + 10$	**c** $-2 + 4$	**d** $-1 + 7$
e $-10 + 14$	**f** $-9 + 6$	**g** $-4 + 8$	**h** $-5 + 5$
i $9 + (-3)$	**j** $4 + (-1)$	**k** $6 + (-4)$	**l** $2 + (-2)$
m $3 + (-8)$	**n** $7 + (-10)$	**o** $5 + (-6)$	**p** $1 + (-9)$

2 Evaluate each difference and check your answer using a calculator.

A **difference** is the answer to a subtraction

a $7 - 9$	**b** $5 - 11$	**c** $3 - 10$	**d** $2 - 6$
e $-4 - 4$	**f** $-8 - 2$	**g** $-1 - 5$	**h** $-3 - 9$
i $6 - (-6)$	**j** $4 - (-1)$	**k** $7 - (-5)$	**l** $10 - (-8)$
m $-5 - (-4)$	**n** $-6 - (-9)$	**o** $-3 - (-3)$	**p** $-1 - (-2)$

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3 At the ski resort, the temperature rose from −2°C to 7°C. What was the increase in temperature? R

4 Rhys' bank account balance was −$44, meaning he was $44 in debt. He deposited $120 into his account. What is his balance now? Select the correct answer **A**, **B**, **C** or **D**. PS R

A $64 **B** $74 **C** $76 **D** $164

5 Evaluate each expression.

a $12 - 3 - 11$ **b** $6 - 10 + 4$ **c** $-18 + 10 - 3$

d $-7 + 3 + 8$ **e** $8 - 15 + (-6)$ **f** $-2 - 12 + 20$

g $3 + (-5) + 1$ **h** $12 - 14 - (-4)$ **i** $-9 + 6 - (-1)$

j $7 - 7 + (-7)$ **k** $-5 + (-5) + (-5)$ **l** $8 - 10 + (-3)$

6 The temperature changed from 6°C to −8°C overnight. What was the change in temperature? Select **A**, **B**, **C** or **D**. R

A 2°C decrease **B** 14°C decrease **C** 14°C increase **D** 2°C increase

7 Copy and fill in the blank for each equation. Use a number line to help you if you need. PS R

a $5 + \_\_\_ = -4$ **b** $-2 + \_\_\_ = 1$ **c** $-4 + \_\_\_ = -9$

d $\_\_\_ + (-7) = -11$ **e** $3 - \_\_\_ = -3$ **f** $-6 - \_\_\_ = -7$

g $-2 - \_\_\_ = 5$ **h** $\_\_\_ - 2 = -6$ **i** $\_\_\_ - (-8) = 0$

8 The table shows the temperature in 4 towns at 2 different times of the morning. Which town had the smallest change in temperature? PS R

Town	5 a.m.	8 a.m.
Jackson Creek	−6°C	−3°C
Smith Valley	−2°C	3°C
Lake Drake	2°C	6°C
Mount Magner	−3°C	4°C

9 A bird dives into a river from a height of 450 cm above the water, travelling at an average speed of 120 cm per second. How far below the water will it be after 4 seconds? PS

10 Copy and complete the blanks for each equation, where at least one of the numbers must be negative. R

a $\_\_\_ + \_\_\_ = 4$ **b** $\_\_\_ + \_\_\_ = -1$ **c** $\_\_\_ + \_\_\_ = 0$

d $\_\_\_ + \_\_\_ = -9$ **e** $\_\_\_ + \_\_\_ = 2$ **f** $\_\_\_ - \_\_\_ = 4$

g $\_\_\_ - \_\_\_ = -8$ **h** $\_\_\_ - \_\_\_ = 9$ **i** $\_\_\_ - \_\_\_ = -1$

11 If a and b are integers, test whether each equation is true or false. R

a $a + b = b + a$ **b** $a - b = b - a$

12 Simplify each algebraic expression by using integers to test them. R

a $a + (-a)$ **b** $a - (-a)$ **c** $a + a$ **d** $a - a$

13 Investigate whether each sum or difference is always positive (+), always negative (−), or could be either positive or negative (E). R

- **a** A larger integer minus a smaller integer
- **b** A positive integer minus a positive integer
- **c** A positive integer plus a positive integer
- **d** A smaller integer minus a larger integer
- **e** A positive integer plus a negative integer
- **f** A negative integer plus a negative integer
- **g** A positive integer minus a negative integer
- **h** A negative integer minus a negative integer

Investigation

Multiplying integers

1 Copy and complete each multiplication.

- **a** $3 \times (-7) = (-7) + (-7) + (-7) =$ ____
- **b** $2 \times (-5) = (-5) + (-5) =$ ____
- **c** $3 \times (-4) = (-4) + (-4) + (-4) =$ ____
- **d** $4 \times (-2) = (-2) + (-2) + (-2) + (-2) =$ ____
- **e** $2 \times (-6) =$ ____
- **f** $3 \times (-3) =$ ____

2 **a** In pairs or as a group activity, copy the multiplication grid onto a large piece of paper and complete the shaded section.

b Complete the first 6 rows.

c Continue the pattern for each **column**.

×	5	4	3	2	1	0	−1	−2	−3	−4	−5
5											
4											
3											
2											
1											
0											
−1											
−2											
−3											
−4											
−5											

3 Use your completed table to simplify each product.

- **a** $4 \times (-3)$
- **b** -3×5
- **c** $-4 \times (-2)$
- **d** $5 \times (-1)$
- **e** $0 \times (-4)$
- **f** -3×3
- **g** $2 \times (-4)$
- **h** $-5 \times (-5)$

4 How do the signs (positive or negative) of the integers in the question affect the sign of the product (answer)? Try to express the rules for multiplying integers in words.

Multiplying and dividing integers 2.03

Multiplying integers

positive × positive = positive
positive × negative = negative
negative × positive = negative
negative × negative = positive

or

If both numbers have the **same** sign, the answer is **positive**.
If both numbers have **different** signs, the answer is **negative**.

⊗	+	−
+	+	−
−	−	+

Because division is the opposite of multiplication, the rules for dividing integers are the same.

Dividing integers

positive ÷ positive = positive
positive ÷ negative = negative
negative ÷ positive = negative
negative ÷ negative = positive

÷	+	−
+	+	−
−	−	+

Example 12

Evaluate each product.

A **product** is the answer to a multiplication

a -3×5 **b** $-6 \times (-9)$ **c** $(-7)^2$

Solution

a $-3 \times 5 = -15$

$(-) \times (+) = (-)$

On a calculator, enter: (−) 3 × 5 =

b $-6 \times (-9) = 54$

$(-) \times (-) = (+)$

(−) 6 × (−) 9 =

c $(-7)^2 = (-7) \times (-7) = 49$

$(-) \times (-) = (+)$

((−) 7) x^2 =

WS What is the integer question?
WS Integer review
Multiplying and dividing integers
Mutiplying integers game
Integers group clues
Integer quiz: Multiplication
WS Integers writing activity

Example 13

Simplify each quotient. A **quotient** is the answer to a division

a $16 \div (-2)$ **b** $-20 \div (-10)$ **c** $\frac{-27}{9}$

Solution

a $16 \div (-2) = -8$

$(+) \div (-) = (-)$

On a calculator, enter: 16 ÷ (–) 2 =.

b $-20 \div (-10) = 2$

$(-) \div (-) = (+)$

(–) 20 ÷ (–) 10 =

c $\frac{-27}{9} = -27 \div 9 = -3$

$(-) \div (+) = (-)$

(–) 27 ÷ 9 =.

EXERCISE 2.03 ANSWERS ON P. 546

Multiplying and dividing integers U F R C

EXAMPLE 12

1 Evaluate each product and check your answer using a calculator.

a	-5×4	**b**	$3 \times (-6)$	**c**	$-4 \times (-8)$	**d**	-9×5
e	$10 \times (-7)$	**f**	$-15 \times (-2)$	**g**	$(-3)^2$	**h**	$6 \times (-4)$
i	-7×3	**j**	$(-10)^2$	**k**	$9 \times (-4)$	**l**	$-8 \times (-5)$
m	$4 \times (-11)$	**n**	-20×3	**o**	$(-6)^2$	**p**	$-7 \times (-9)$
q	-11×5	**r**	$(-9)^2$	**s**	$10 \times (-10)$	**t**	$-6 \times (-7)$

2 For each product, select the correct answer **A**, **B**, **C** or **D**.

a $12 \times (-3)$

A 15 **B** -15 **C** -36 **D** 36

b $-3 \times 6 \times 2$

A 0 **B** -16 **C** -18 **D** -36

c $5 \times 3 \times (-2)$

A -15 **B** -30 **C** 15 **D** 30

EXAMPLE 13

3 Evaluate each quotient and check your answer using a calculator.

a	$36 \div (-4)$	**b**	$-15 \div (-3)$	**c**	$-14 \div 2$	**d**	$60 \div (-10)$
e	$-25 \div (-5)$	**f**	$-28 \div 4$	**g**	$45 \div (-9)$	**h**	$-32 \div 16$
i	$-36 \div (-6)$	**j**	$56 \div (-7)$	**k**	$-24 \div 3$	**l**	$-42 \div (-6)$
m	$40 \div (-8)$	**n**	$-20 \div (-1)$	**o**	$-81 \div 9$	**p**	$24 \div (-4)$

4 Evaluate each quotient. Select **A**, **B**, **C** or **D**.

a $-45 \div (-5)$

A 9 **B** -9 **C** -50 **D** -40

b $\frac{42}{-6}$

A 36 **B** -7 **C** 7 **D** -48

5 Evaluate each quotient.

a $\frac{-18}{2}$ **b** $\frac{16}{-4}$ **c** $\frac{-30}{-5}$ **d** $\frac{21}{-3}$

e $\frac{-32}{-8}$ **f** $\frac{-100}{25}$ **g** $\frac{-81}{-9}$ **h** $\frac{56}{-8}$

6 Copy and complete each equation. R

a $7 \times \_\_\_ = -14$ **b** $-3 \times \_\_\_ = 12$ **c** $-5 \times \_\_\_ = -25$

d $\_\_\_\_ \times 3 = -21$ **e** $\_\_\_\_\_ \times (-4) = 28$ **f** $\_\_\_\_ \times 9 = -63$

7 Evaluate each product and check your answer using a calculator.

a $-2 \times (-2) \times 7$ **b** $6 \times 5 \times (-3)$ **c** $4 \times (-1) \times 10$

d $3 \times (-5)^2$ **e** $-5 \times 2 \times (-2)$ **f** $(-2)^3$

8 Evaluate $-2 + 7 \times (-3) + 12$. Select **A**, **B**, **C** or **D**.

A -3 **B** -11 **C** 31 **D** 61

9 Evaluate each expression and check your answer using a calculator.

a $-28 \div 2 \div 7$ **b** $45 \div (-3) \div (-5)$ **c** $36 \div (-9) \div 2$

d $-90 \div 3 \div (-1)$ **e** $100 \div 2 \div (-10)$ **f** $-24 \div (-6) \div (-4)$

g $8 \times (-5) \div 2$ **h** $-4 \times 7 \div 14$ **i** $35 \div (-5) \times 10$

10 Copy and complete each equation. R

a $-12 \div \_\_\_ = 2$ **b** $15 \div \_\_\_ = -3$ **c** $-9 \div \_\_\_ = -1$

d $\_\_\_\_ \div (-4) = -8$ **e** $\_\_\_\_ \div 12 = -2$ **f** $\_\_\_\_ \div (-10) = -10$

11 Evaluate $4 - 24 \div (-6) + 7$. Select **A**, **B**, **C** or **D**.

A -20 **B** 7 **C** 10 **D** 15

12 If a and b are integers, test whether each equation is true or false. R

a $a \times b = b \times a$ **b** $a \div b = b \div a$

13 What happens when you multiply an integer by its opposite? R C

Foundation Standard Complex

14 Investigate whether each product is always positive (+), always negative (−), or could be either positive or negative (E). R

a The product of 2 positive integers

b The product of 2 negative integers

c The product of a positive integer and a negative integer

d A positive integer squared

e A negative integer squared

15 Copy and complete for each equation, where at least one of the numbers must be negative. R

a ___ × ___ = 20 **b** ___ × ___ = −16 **c** ___ × ___ = 36

d ___ × ___ = −5 **e** ___ × ___ = 9 **f** ___ × ___ = −22

16 What happens when you divide an integer: R C

a by itself? **b** by its opposite?

17 Simplify each algebraic expression by using integers to test them. R

a $a \times a$ **b** $(-a) \times (-a)$ **c** $a \div a$ **d** $(-a) \div a$

18 Copy and complete each equation, where at least one of the numbers must be negative. R

a ___ ÷ ___ = 9 **b** ___ ÷ ___ = −5 **c** ___ ÷ ___ = 3

d ___ ÷ ___ = −4 **e** ___ ÷ ___ = 10 **f** ___ ÷ ___ = −7

Did you know?

Pascal's triangle

Pascal's triangle is a number pattern named after the French mathematician Blaise Pascal (1623 – 1662) but it was known about much earlier.

Each number is found by adding the 2 numbers just to the left and right of it in the previous line.

There are many patterns in this triangle:

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

- The sum of each row are the powers of 2
- The Fibonacci numbers are found by summing minor diagonals
- The counting numbers and triangular numbers are found along major diagonals

The triangle has these simple patterns but also provides much more complex patterns, such as binomial expansions in algebra and probability.

Use the Internet to research the pattern in the triangle that gives the powers of 11.

Find 2 other patterns in Pascal's triangle.

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9780170453141

Mental skills 2A: Maths without calculators ANSWERS ON P. 546

Multiplying decimals

1 Study each example.

a $3 \times 8 = 24$, so $3 \times 0.8 = 2.4$

0 dp + 1 dp = 1 dp (dp = decimal places)

The number of decimal places in the answer is equal to the total number of decimal places in the question. Also, the answer sounds reasonable because, by estimation:

$3 \times 0.8 \approx 3 \times 1 = 3$ $(2.4 \approx 3)$

b $6 \times 5 = 30$, so $0.6 \times 0.5 = 0.30 = 0.3$

1 dp + 1 dp = 2 dp

By estimation, $0.6 \times 0.5 \approx 0.5 \times 0.5 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = 0.25$ $(0.3 \approx 0.25)$

c $7 \times 3 = 21$, so $0.07 \times 0.3 = 0.021$

2 dp + 1 dp = 3 dp

By estimation, $0.07 \times 0.3 \approx 0.07 \times \frac{1}{3} \approx 0.02$ $(0.021 \approx 0.02)$

2 Now evaluate each product.

a	0.7×5	**b**	12×0.2	**c**	0.4×0.3	**d**	$(0.6)^2$
e	8×0.1	**f**	0.03×0.9	**g**	4×0.05	**h**	1.1×8
i	0.3×0.8	**j**	0.2×0.06	**k**	9×0.2	**l**	0.07×0.4

3 Study each example.

Given that $15 \times 23 = 345$, evaluate each product.

a $1.5 \times 2.3 = 3.45$

1 dp + 1 dp = 2 dp (*Estimate*: $1.5 \times 2.3 \approx 2 \times 2 = 4$)

b $150 \times 0.23 = 15 \times 10 \times 0.23 = 15 \times 0.23 \times 10 = 3.45 \times 10 = 34.5$

0 dp + 2 dp = 2 dp

(*Estimate*: $150 \times 0.23 \approx 150 \times 0.2 = 150 \times \frac{1}{5} = 30$)

c $0.15 \times 2300 = 0.15 \times 23 \times 100 = 3.45 \times 100 = 345$

2 dp + 0 dp = 2 dp

(*Estimate*: $0.15 \times 2300 \approx 0.2 \times 2300 = \frac{1}{5} \times 2300 = 460$)

4 Now given that $39 \times 17 = 663$, evaluate each product.

a	3.9×17	**b**	39×170	**c**	39×0.17	**d**	0.39×1.7
e	3.9×1.7	**f**	390×1.7	**g**	3.9×0.17	**h**	3.9×170
i	3900×1.7	**j**	39×1.7	**k**	39×0.017	**l**	0.39×0.17

2.04 Order of operations

Order of operations

Order of operations

Order of operations

BODMAS

Directed numbers

Order of operations puzzle

When evaluating a mixed expression such as $18 \div (-2 + 1) \times 2$, there is a specific order in which the different operations are performed.

Order of operations

1. Simplify any expression inside grouping symbols (brackets): start with the innermost brackets first.
2. Simplify any multiplication ($\times$) and division ($\div$), from left to right.
3. Simplify any addition ($+$) and subtraction ($-$), from left to right.

Beware of cheap calculators that do not follow the 'order of operations' rules!

Example 14

Evaluate each expression.

a $5 \times 2 + 3 \times (-9)$ **b** $2 \times [(25 - 4) \div 3]$

Solution

a $5 \times 2 + 3 \times (-9) = 10 + (-27)$
$= -17$

Multiply first: $5 \times 2 = 10$, and $3 \times (-9) = -27$
On a calculator, enter:
5 × 2 + 3 × (–) 9 =.

b $2 \times [(25 - 4) \div 3] = 2 \times [21 \div 3]$
$= 2 \times 7$
$= 14$

Innermost brackets first: $25 - 4 = 21$
Next brackets: $21 \div 3 = 7$
2 × ((25 − 4) ÷ 3) =.

Example 15

Simplify each fraction.

a $\frac{8+16}{40 \div 10}$ **b** $\frac{10-30}{6^2}$

Solution

To simplify each fraction, evaluate the numerator and denominator separately, then divide the numerator by the denominator.

a $\frac{8+16}{40\div10} = \frac{24}{4}$

$= 6$

On a calculator, enter: (8 + 16) ▭ (40 ÷ 10) =

OR if using MATH Mode, press ▭ and enter 8 + 16 and 40 ÷ 10 separately into the blank spaces.

b $\frac{10-30}{6^2} = \frac{-20}{36}$

$= -\frac{5}{9}$

Simplifying the fraction.

EXERCISE 2.04 ANSWERS ON P. 546

Order of operations U F PS R C

1 Evaluate each expression and check your answer on a calculator.

EXAMPLE 14

a	$15 \div (7-4)$	**b**	$[14+(-9)] \times 6$	**c**	$20-(5-7)$
d	$8 \times (-3+5-6)$	**e**	$-2 \times (10-9)+28$	**f**	$(16+8) \div 2-4$
g	$18 \div (5-2) \times (-2)$	**h**	$[4-(-7)] \times 3-10$	**i**	$-26 \div (14+12)$
j	$(7-10) \times 10 \div 6$	**k**	$5 \times [(22-10) \div (-3)]+1$	**l**	$30+[7 \times (2-6)]-2$

2 Evaluate each expression and check your answer on a calculator.

a	$8+5 \times (-2)$	**b**	$7-2 \times 3$	**c**	$6 \times 5-(-1)$
d	$12+(-6) \div 3$	**e**	$3 \times 6+2 \times 5$	**f**	$9-11+15 \div 3$
g	$-5 \times 10+16 \div 2$	**h**	$-3 \times 6-(-2) \times 5$	**i**	$42 \div 6+9$
j	$17+8-3 \times 2$	**k**	$4 \times 3-(-7)+5$	**l**	$16-3 \times (-4) \div 2$

3 Simplify each fraction.

EXAMPLE 15

a	$\frac{5\times2}{16+4}$	**b**	$\frac{19+5}{18-6}$	**c**	$\frac{9\times3}{40-10}$	**d**	$\frac{58+8}{-7-4}$
e	$\frac{4^2}{16\div(-2)}$	**f**	$\frac{5\times3-1}{16+10\times4}$	**g**	$\frac{28-5\times3}{(56-30)\div2}$	**h**	$\frac{25\div5+13}{\sqrt{36}}$

4 Which expression is equal to 4? Select the correct answer **A**, **B**, **C** or **D**.

A	$20 \div 4+6 \times 2$	**B**	$20 \div (4+6) \times 2$
C	$20 \div [(4+6) \times 2]$	**D**	$(20 \div 4+6) \times 2$

5 Ms Ferme, the Mathematics Head Teacher, has a parent complaining about the marking of his daughter's maths exam. He claims his daughter's correct answer was marked wrong.

This is the question: $-9+13 \times 4$

His daughter's answer was 16. R C

a Why was the daughter's answer wrong?

b How could Ms Ferme explain why the daughter's answer is wrong?

c Put brackets in $-9+13 \times 4$ to make his daughter's answer correct.

Foundation | Standard | Complex

6 Copy and complete each equation. R

a $-8 \times 3 - \_\_\_ = -32$
b $\_\_\_ \times (5 - 9) = -20$
c $(8 - \_\_\_) \times 7 = 35$
d $9 \times \_\_\_ + (-2) = -20$
e $24 \div (-3) \times \_\_\_ = 8$
f $\_\_\_ + 5 \times 3 = -5$
g $(-2 - \_\_\_) \times (-4) = -16$
h $(-3 - 9) \div \_\_ = -2$
i $\_\_\_ \div (-4) + 6 = 1$

7 Copy each equation and insert grouping symbols in the correct places to make the equation true. R

a $8 - 3 + 7 = -2$
b $40 - 10 \times 5 = 150$
c $27 \div 9 \div 3 = 9$
d $8 + 4 - 3 \times 2 = 10$
e $8 + 4 - 3 \times 2 = 18$
f $6 + 4 \times (-1) - 1 = -11$
g $13 + 3 \div 4 - 6 = -8$
h $100 \div 10 + 10 + 5 = 10$
i $100 \div 10 + 10 + 5 = 4$

8 Use all the integers 5, −2 and 10 and grouping symbols to complete each equation. PS R

a _____ = 30
b _____ = −1
c _____ = −60

Spread-sheets

Review of spreadsheets

Technology

Order of operations

1 Enter these 7 values in column A of a new spreadsheet: 36, 7, 8.5, 18, 50, 2, −3.

2 In column B, write an appropriate formula to evaluate each expression in the given cell. Try to predict the answer before you enter each formula. The first one has been done for you.

B1: $36 + 7 - 2$ In cell B1, enter **=A1+A2−A6**

B2: $(7 + 8.5) \div 2$
B3: $7 + 8.5 \div 2$
B4: $50 - 2 \times (-3)$
B5: $18 \times 7 + 36 \times 8.5$
B6: $18 \times (7 + 36) \times 8.5$
B7: $\frac{50}{2+18}$
B8: $50 + 8.5 - 2 - (-3)$
B9: $7 \times 2 + 50 \div 2$
B10: $\frac{50 \times (2+18+8.5)}{-3}$

2.05 Decimals

Decimals

Decimals wall

Dewey decimals

Ordering decimals

- List them in a column with the decimal points in line so that the place values can be compared
- Fill any gaps at the end with 0s
- **'Ascending order'** means going up, from smallest to largest
- **'Descending order'** means going down, from largest to smallest.

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Example 16

Write these decimals in ascending order: 3.5, 3.751, 3.15, 3.157.

Solution

3.500 — List the numbers in a column with the decimal points aligned.
3.751 — Fill any gaps at the end with zeros.
3.150 — Order the decimals from smallest to largest by comparing place values.
3.157 — The smallest is 3.150, next is 3.157, next is 3.500, the largest is 3.751.

In ascending order, the decimals are 3.15, 3.157, 3.5, 3.751.

Rounding decimals

To **round a decimal**, cut it at the required decimal place and look at the digit in the next place:

- if that digit is less than 5 (that is, 0, 1, 2, 3 or 4), **round down**
- if the digit is 5 or more (that is, 5, 6,7, 8 or 9), **round up**.

Example 17

a Round 5.261 to the nearest tenth.

b Write 14.8239 correct to 2 decimal places.

Solution

a 'nearest tenth' = to one decimal place

5.2 | 61

cut — the next digit is 6 (greater than 5), so round **up** to 5.3

$5.261 \approx 5.3$

b 14.82 | 39

cut — the next digit is 3 (less than 5), so round **down** to 14.82

$14.8239 \approx 14.82$

Adding and subtracting decimals

When **adding and subtracting decimals**, keep decimal points below one another.

Fill any gaps with 0s.

Decimals 1

Decimals 10

2.05

Rounding decimals

Example 18

Evaluate each expression.

a $2.7 + 44.3 + 13.25$ **b** $23.8 - 5.65$

Solution

a

$$\begin{array}{r} {\scriptstyle 1\ \ 1\ \ \ \ } \\ 2.70 \\ 44.30 \\ +13.25 \\ \hline 60.25 \end{array}$$

Line up decimal points in the same column.
Fill any gap with 0s.

Check by estimating: $2.7 + 44.3 + 13.25 \approx 3 + 44 + 10 = 57$ (60.25 is close to 57)

b

$$\begin{array}{r} {\scriptstyle 1\ 13\ \ 7\ \ 10} \\ 2\not{3}.\not{8}\not{0} \\ -5.65 \\ \hline 18.15 \end{array}$$

Line up decimal points in the same column.
Fill any gap with 0s.

Check by estimating:
$23.8 - 5.65 \approx 24 - 6$
$= 18$ (18.15 is close to 18)

EXERCISE 2.05 ANSWERS ON P. 546

Decimals U F PS R

EXAMPLE 16

1 Which decimal is the largest? Select the correct answer **A**, **B**, **C** or **D**.

A 6.408 **B** 6.48 **C** 6.048 **D** 6.4

2 Write each set of decimals in ascending order.

a 4.35, 4.3, 4.05, 4.035 **b** 17.2103, 17.12, 12.173, 17.231

c 14.8, 14.08, 14.801, 14.81 **d** 0.3, 0.295, 0.032, 0.2

3 Write each set of decimals in descending order.

a 1.612, 1.621, 1.61, 1.16 **b** 3.05, 3.053, 3.035, 3.305

4 How many decimal places has each decimal?

a 3.824 **b** 1.09 **c** 51.6 **d** 22

e 12.573 **f** 450.304 **g** 788.5 **h** 0.45

EXAMPLE 17

5 Write each decimal correct to one decimal place.

a 3.851 **b** 4.0736 **c** 0.3333 **d** 7.34

e 15.0801 **f** 2.976 **g** 2.048 **h** 16.1919

6 What is 12.3752 rounded to the nearest hundredth? Select **A**, **B**, **C** or **D**.

A 12.8 **B** 12.37 **C** 12.38 **D** 12.375

7 Round each decimal to the nearest hundredth.

a 68.9109 **b** 107.0594 **c** 3.5963 **d** 4.7077

e 3.198 **f** 32.999 **g** 19.7291 **h** 11.254

8 Write each decimal correct to 3 decimal places.

a 9.7043 **b** 13.167 54 **c** 0.08281 **d** 53.094 23

9 Round $460.39512 to the nearest:

a cent **b** dollar **c** tenth **d** 5 cents **e** thousandth

10 Evaluate each expression, then check your answer with an estimate.

a $28.51 + 136.4$ **b** $7.2 + 18.16$

c $34.7 + 29.4 + 8.5$ **d** $127.81 - 36.2$

e $46.5 - 30.8$ **f** $100.87 - 23.6$

g $20.03 - 1.59$ **h** $12.56 - 9.88$

i $71.5 - 4.82$ **j** $65.21 - 13.6$

k $9 - 4.36$ **l** $3.671 - 1.28$

EXAMPLE 18

Shutterstock.com/Martin Allinger

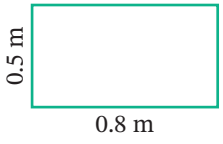
11 Evaluate $0.52 + 14.025$. Select **A**, **B**, **C** or **D**.

A 65.025 **B** 19.225

C 14.545 **D** 14.077

12 Yesterday, the temperature dropped from 24.1°C to 16.5°C. What was the difference between the 2 temperatures?

13 Monique built a wooden frame with dimensions 0.8 m by 0.5 m. How much wood will be left from a 3.4 m length of timber? PS



14 Tahir bought the following items: an exercise book for $2.70, 2 pens for $1.60 each, a drink for $1.50 and a packet of chips for $2.65. PS

a How much did Tahir spend in total?

b If he paid with a $20 note, how much change did he receive?

15 Copy and complete each equation with the correct decimals. R

a $2.8 + \_\_\_\_\_ = 5.84$ **b** $\_\_\_\_\_ - 7.6 = 5.14$

c $18.75 - \_\_\_\_ = 13.3$ **d** $\_\_\_\_\_ + 0.83 = 1.719$

e $\_\_\_\_\_ - 1.07 = 3.256$ **f** $\_\_\_\_\_ - \_\_\_\_\_ = 4.89$

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Investigation

Rounding up or down?

Sometimes, the decision on whether to round an answer up or down depends on the situation. In groups of 2 to 4, discuss each of the following situations and decide whether it is more appropriate to round up or to round down. You must give a reason for your choice.

1 7 friends have dinner at a restaurant and the total bill is $206. They decide to share the bill evenly: $206 ÷ 7 = $29.428 57...

How much should each friend pay, to the nearest dollar: $29 or $30?

Shutterstock.com/G-Stock Studio

2 You have a budget of $65 for buying drinks for a party. One can of drink costs $1.74 at the supermarket: $65 ÷ $1.74 = 37.356 32...

How many whole cans of drink can you buy: 37 or 38?

3 You need to find the average number of people living in each home in your street. You survey 48 homes and count a total of 166 people: 166 ÷ 48 = 3.458 33...

What is the average whole number of people living in each house: 3 or 4?

4 You need to paint the walls of a house with a total surface area of 334 m^2. One tin of paint covers 64 m^2: 334 ÷ 64 = 5.218 75

How many whole tins of paint do you need for the job: 5 or 6?

5 Jodie has $2098 in her bank account and the bank pays her 4.71% in interest:

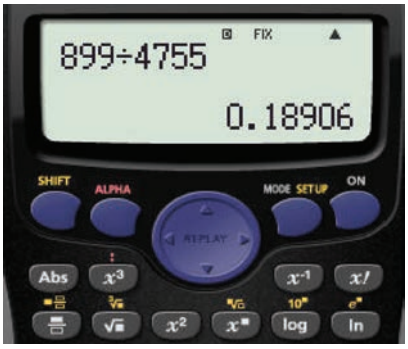
4.71% × $2098 = $98.8158...

How much interest will the bank pay Jodie: $98.81 or $98.82?

Technology

The FIX mode on a calculator

Most scientific calculators have a FIX mode that rounds a displayed answer to a given number of decimal places. However, the exact answer is still stored in the calculator's memory to make sure that any further calculations involving this answer are accurate.
Find out how to use the FIX mode on your calculator.
The FIX mode has been used to set this calculator to display answers to 5 decimal places.



Multiplying and dividing decimals 2.06

Multiplying decimals

When multiplying decimals, the number of decimal places in the answer equals the *total* number of decimal places in the question.

Example 19

Evaluate each product.

a 0.8×0.3 **b** 7.2×5 **c** 5.31×1.3

Solution

a $8 \times 3 = 24$ — Multiply without decimal points first.

Count the number of decimal places in the question. — 0.8 has one decimal place, 0.3 has one decimal place.

$0.8 \times 0.3 = 0.24$ — Write the answer with $1 + 1 = 2$ decimal places.

Check by estimating: $0.8 \times 0.3 \approx 1 \times 0.3 = 0.3$ $(0.24 \approx 0.3)$

b $72 \times 5 = 360$ — Multiply without decimal points first.

7.2 has one decimal place, 5 has no decimal places.

$7.2 \times 5 = 36.0$
$= 36$ — Write the answer with $1 + 0 = 1$ decimal place.

Check by estimating: $7.2 \times 5 \approx 7 \times 5 = 35$ $(36 \approx 35)$

Multiplying decimals
Decimal cards
Shopping and change
Where's the point?
Decimals 7
Operations with decimals
Which decimals?
Decimal number grids

Movie night

c

$$\begin{array}{r} 531 \\ \times\ 13 \\ \hline 1593 \\ 5310 \\ \hline 6903 \end{array}$$

Multiply without decimal points first.

5.31 has 2 decimal places, and 1.3 has 1 decimal place.

$5.31 \times 1.3 = 6.903$

Write the answer with $2 + 1 = 3$ decimal places.

Check by estimating: $5.31 \times 1.3 \approx 5 \times 1 = 5$

$(6.903 \approx 5)$

Dividing a decimal by a whole number

- Rewrite the question in 'short division' form
- Make the decimal point in the answer line up with the decimal point in the question
- Add 0s to the end of the decimal being divided, if needed

Dividing decimals

Dividing a decimal by another decimal

- Make the second decimal a whole number by moving the decimal point the required number of places to the right
- Move the point in the first decimal the same number of places to the right
- Divide the new first number by the whole number

This works because we are multiplying both decimals by the same power of 10 before dividing.

Example 20

Evaluate each quotient.

a $6.28 \div 5$ **b** $12.4 \div 0.04$ **c** $4.98 \div 1.2$

Solution

a

$$5\overline{)6.^{1}2^{2}8^{3}0}\quad 1.\ 2\ 5\ 6$$

Write $6.28 \div 5$ as a short division.

- 5 into 6 goes 1, remainder 1
- Make the decimal point in the answer line up with the decimal point in the question
- 5 into 12 goes 2, remainder 2.
- 5 into 28 goes 5, remainder 3.
- Add a zero to 6.28 so that you can complete the division.
- 5 into 30 goes 6 exactly.

$6.28 \div 5 = 1.256$.

Check by estimating: $6.28 \div 5 \approx 6 \div 5 \approx 1$

$(1.256 \approx 1)$

b $12.4 \div 0.04 = 1240 \div 0\,04$

$= 1240 \div 4$

$= 310$

Move both decimal points 2 places to the right so that 0.04 becomes a whole number.

We added a 0 to the end of 12.4 so that we can move the point 2 places to the right.

Check by estimating: $1240 \div 4 \approx 1200 \div 4 \approx 300$ ($310 \approx 300$)

c $4.98 \div 1.2 = 49.8 \div 12$

Move both decimal points one place to the right so that 1.2 becomes a whole number.

$$\begin{array}{r} 4.15 \\ 12\overline{)49.80} \\ -48\downarrow \\ \hline 18 \\ -12 \\ \hline 60 \\ -60 \\ \hline 0 \end{array}$$

Write $49.8 \div 12$ as a long division:

12 into 49 is 4, remainder 1

12 into 18 is 1, remainder 6

12 into 60 is 5 exactly

$4.98 \div 1.2 = 4.15$

Check by estimating: $4.98 \div 1.2 \approx 5 \div 1 = 5$ ($4.15 \approx 5$)

EXERCISE 2.06 ANSWERS ON P. 547

Multiplying and dividing decimals U F PS R

1 Evaluate each product, then check your answer with an estimate. EXAMPLE 19

a 6.5×4	**b** 0.45×9	**c** 2.3×5	**d** 7×3.9
e 0.7×0.3	**f** 0.8×0.11	**g** 0.05×0.4	**h** 4.2×0.3
i 6.3×4.5	**j** 28.7×3.1	**k** 1.3×0.62	**l** 0.72×0.51

2 Phillip was building a fence. He needed 9.4 metres of wire at $5.80 per metre. What was the cost of the wire? Select the correct answer **A**, **B**, **C** or **D**. PS

A $15.20 **B** $54.52 **C** $152.00 **D** $545.20

3 Thao's mobile phone plan charges $20 per month plus $0.18 for each phone call. How much will Thao need to pay if she made 112 calls in one month? PS

4 Evaluate each quotient, then check your answer with an estimate. EXAMPLE 20

a $24.8 \div 4$	**b** $114.1 \div 7$	**c** $7.83 \div 3$
d $0.695 \div 5$	**e** $63.3 \div 0.2$	**f** $4.173 \div 0.06$
g $18.5 \div 0.05$	**h** $23.22 \div 0.4$	**i** $14.5 \div 25$
j $6.24 \div 1.2$	**k** $238 \div 1.4$	**l** $0.252 \div 0.21$

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5 What is the value of $12.92 \div 0.04$? Select **A**, **B**, **C** or **D**.

A 32.3 **B** 323 **C** 3.23 **D** 3230

6 Brad uses 7.24 metres of wood to make a garden bed fence. PS R

Shutterstock.com/Paul Maguire

a How much wood does he need to make 6 garden bed fences?

b If one of the garden beds is square-shaped, how long is each side?

7 Evaluate each expression.

a $0.3 \times 0.7 + 5.7$ **b** $2.8 \div (7.4 - 7.2)$ **c** $3.62 + 4.08 \div 2$

8 Jordan's car holds 45 litres of petrol. If the price of petrol is 149.9 cents per litre, how much will Jordan need to pay to fill the tank? Give your answer to the nearest 5 cents. PS

9 Copy and complete each equation with the correct decimals. R

a $0.3 \times$ _____ $= 2.4$ **b** _____ $\times 0.6 = 1.2$ **c** _____ $\times 0.7 = 0.028$

d $3.1 \times$ _____ $= 2.79$ **e** $83.4 \div$ _____ $= 6$ **f** _____ $\div 8.1 = 3.2$

10 A car travels 145.8 kilometres on 9 litres of petrol. How many kilometres could it travel on one litre of petrol? PS

11 Anja earned $169.20 for working a 12-hour shift. How much was earned each hour? PS

12 Henry has a faulty calculator that does not show the decimal point. For each calculation in the table shown, write the correct answer. PS R

	Calculation	Answer with missing decimal point
a	3.42 × 12	4104
b	4.145 × 0.2	829
c	37.3 × 8.8	32824
d	0.03 × 157.64	47292
e	8.3902 × 0.3	251706

Terminating and recurring decimals

2.07

When converting a fraction to a decimal, or dividing 2 numbers, the decimal answer can be **terminating** or **recurring**.

Terminating decimals, such as 0.625, have a definite number of decimal places, while **recurring decimals**, such as 0.272727..., written as $0.\dot{2}\dot{7}$ or $0.\overline{27}$, have one or more digits that repeat endlessly. We use dots or lines to mark the repeating section. For example, 0.259 259 259... $= 0.\dot{2}5\dot{9}$ or $0.\overline{259}$.

'Terminate' means 'to stop', while 'recurring' means 'repeating'.

Fractions and decimals
Fraction families
Decimals squaresaw 2
Converting fractions to decimals

Example 21

Convert each fraction to a recurring decimal.

a $\frac{5}{6}$ **b** $\frac{2}{11}$

Solution

a $\frac{5}{6}$ means $5 \div 6$

$$6\overline{)5.0000\ldots}\quad 0.8333\ldots$$

Add 0s to complete the division.

A recurring decimal.

$\frac{5}{6} = 0.8333... = 0.8\dot{3}$ or $0.8\overline{3}$

b $\frac{2}{11}$ means $2 \div 11$

$$11\overline{)2.00000\ldots}\quad 0.18181\ldots$$

A recurring decimal.

$\frac{2}{11} = 0.181\,818... = 0.\dot{1}\dot{8}$ or $0.\overline{18}$

Example 22

Evaluate each quotient.

a $2.8 \div 9$ **b** $0.7 \div 0.12$

Solution

a $2.8 \div 9$

$$9\overline{)2.8000\ldots}\quad 0.3111\ldots$$

A recurring decimal.

$2.8 \div 9 = 0.3111... = 0.3\dot{1}$ or $0.3\overline{1}$

b $0.7 \div 0.12 = 70 \div 12$

$$12\overline{)70.0000\ldots}\quad 5.8333\ldots$$

A recurring decimal.

$0.7 \div 0.12 = 5.83333\ldots = 5.8\dot{3}$ or $5.8\overline{3}$

EXERCISE 2.07 ANSWERS ON P. 547

Terminating and recurring decimals U F R C

1 Convert each fraction to a terminating decimal.

a $\frac{2}{5}$ **b** $\frac{3}{8}$ **c** $\frac{3}{4}$ **d** $\frac{2}{2}$

e $\frac{2}{4}$ **f** $\frac{6}{8}$ **g** $\frac{3}{5}$ **h** $\frac{2}{8}$

i $\frac{5}{8}$ **j** $\frac{7}{8}$ **k** $\frac{4}{8}$ **l** $\frac{5}{5}$

2 Explain why some of the fractions in question **1** have the same decimal value. R C

3 Rewrite each recurring decimal using dot notation. C

a 0.6666... **b** 4.2727... **c** 0.2828... **d** 3.8333...

e 9.607607... **f** 0.15191519... **g** 0.05252... **h** 12.23444...

4 Write each recurring decimal showing the repeated pattern. R C

a $0.858\dot{3}$ **b** $0.16\dot{5}\dot{5}$ **c** $0.\dot{2}\dot{7}$ **d** $0.\dot{4}61538\dot{4}$

EXAMPLE 21

5 Convert each fraction to a recurring decimal. C

a $\frac{1}{9}$ **b** $\frac{1}{6}$ **c** $\frac{5}{6}$ **d** $\frac{1}{7}$ **e** $\frac{2}{3}$

f $\frac{2}{7}$ **g** $\frac{2}{9}$ **h** $\frac{3}{7}$ **i** $\frac{4}{7}$ **j** $\frac{4}{9}$

k $\frac{4}{6}$ **l** $\frac{5}{9}$ **m** $\frac{6}{7}$ **n** $\frac{7}{9}$ **o** $\frac{5}{7}$

6 Copy and complete the table and note the pattern.

Fraction	$\frac{1}{7}$	$\frac{2}{7}$	$\frac{3}{7}$	$\frac{4}{7}$	$\frac{5}{7}$	$\frac{6}{7}$
Decimal						

EXAMPLE 22

7 Evaluate each expression as a recurring decimal.

a $11 \div 3$ **b** $15.4 \div 9$ **c** $58.43 \div 0.11$ **d** $1.96 \div 0.6$

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Mental skills 2B: Maths without calculators ANSWERS ON P. 547

Dividing decimals

To divide one decimal by another, first move the decimal points in **both** decimals the same number of places to the right so that the second decimal is a **whole number**.

1 Study each example.

a $0.24 \div 0.06 = 24 \div 6 = 4$

b $0.45 \div 0.5 = 4.5 \div 5 = 0.9$

c $0.006 \div 0.3 = 0.06 \div 3 = 0.02$

d $27 \div 0.9 = 270 \div 9 = 30$

e $1.6 \div 0.4 = 16 \div 4 = 4$

f $5.6 \div 0.07 = 560 \div 7 = 80$

2 Now evaluate each quotient.

a $0.25 \div 0.5$

b $63 \div 0.7$

c $3.2 \div 0.4$

d $0.18 \div 0.2$

e $2.7 \div 0.03$

f $0.042 \div 0.06$

g $4 \div 0.5$

h $1.2 \div 0.04$

i $0.072 \div 0.9$

j $0.35 \div 0.1$

k $0.28 \div 0.07$

l $0.033 \div 0.11$

3 Study each example.

Given that $112 \div 14 = 8$, evaluate each expression.

a $112 \div 1.4 = 112.0 \div 1.4$
$= 1120 \div 14$
$= 112 \times 10 \div 14$
$= 112 \div 14 \times 10$
$= 8 \times 10$
$= 80$
Estimate: $112 \div 1.4 \approx 112 \div 1 = 112$

b $0.112 \div 0.14 = 0.112 \div 1.4$
$= 11.2 \div 14$
$= 112 \div 10 \div 14$
$= 112 \div 14 \div 10$
$= 8 \div 10$
$= 0.8$
Estimate: $0.112 \div 0.14 \approx 0.1 \div 0.1 = 1$

c $1120 \div 1.4 = 11200 \div 14$
$= 112 \times 100 \div 14$
$= 112 \div 14 \times 100$
$= 8 \times 100$
$= 800$
Estimate: $1120 \div 1.4 \approx 1120 \div 1 = 1120$

d $1.12 \div 14 = 112 \div 100 \div 14$
$= 112 \div 14 \div 100$
$= 8 \div 100$
$= 0.08$
Estimate: $1.12 \div 14 \approx 1.12 \div 10 = 0.112$

4 Now given that $368 \div 23 = 16$, evaluate each quotient.

a $36.8 \div 2.3$

b $368 \div 2.3$

c $3.68 \div 2.3$

d $0.368 \div 0.23$

e $36.8 \div 23$

f $3.68 \div 0.23$

g $36.8 \div 0.23$

h $0.368 \div 2.3$

i $0.368 \div 23$

j $3.68 \div 0.023$

k $3.68 \div 23$

l $0.368 \div 230$

2.08 Powers and roots

Index notation

Square roots and cube roots

Powers and roots

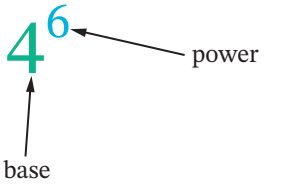
Big numbers

Square root Snap

Powers and the use of **index notation** allow us to write repeated multiplication in a shorter way.

$4^6 = 4 \times 4 \times 4 \times 4 \times 4 \times 4$ '4 to the power of 6'

In 4^6, the number 4 is called the **base** and is the number that is repeated in the multiplication. The small raised number 6 is called the **power** or **index**.



The power keys on a calculator

- The square of a number is found using the x^2 key
- The cube of a number is found using the x^3 key
- Any power of a number can be found using the $x^{\blacksquare}$ or y^x key.

Example 23

Evaluate each power.

a 11^3 **b** $(-8)^4$

Solution

a $11^3 = 1331$ On a calculator, enter: 11 x^3 =

b $(-8)^4 = 4096$ ((–) 8) $x^{\blacksquare}$ 4 =

Square root and cube root

- The **square root** ($\sqrt{\ }$) of a number is the **positive** value which, if squared, will give that number.
- The **cube root** ($\sqrt[3]{\ }$) of a given number is the positive or negative value which, if cubed, will give that number.

For example:

$\sqrt{36} = 6$	because $6^2 = 36$	'the square root of 36'
$\sqrt[3]{125} = 5$	because $5^3 = 125$	'the cube root of 125'
$\sqrt[3]{-8} = -2$	because $(-2)^3 = -8$	'the cube root of − 8'

Most roots do not give exact answers like the ones above, and are called **surds**. For example, $\sqrt{7} = 2.645751311... \approx 2.6$. A surd is a square root ($\sqrt{\ }$), cube root ($\sqrt[3]{\ }$), or any other type of root whose exact decimal or fraction value cannot be found. As a decimal, its digits run endlessly *without repeating*, so they are *neither* terminating nor recurring decimals. A surd cannot be written in fraction form $\frac{a}{b}$, so it is also called an **irrational** number.

Example 24

Evaluate each cube root, correct to 2 decimal places where necessary.

a $\sqrt[3]{729}$ **b** $\sqrt[3]{100}$

Solution

a $\sqrt[3]{729} = 9$ On a calculator, enter: $\sqrt[3]{\ }$ 729 =

b $\sqrt[3]{100} = 4.6415 \ldots \approx 4.64$ $\sqrt[3]{\ }$ 100 =

Example 25

Estimate the value of $\sqrt{50}$, correct to one decimal place.

Solution

There is no *exact* answer for the square root of 50, because there isn't a number which, if squared, equals 50 exactly. However, we can find a decimal whose square is close to 50.

Noting that:

$6^2 = 36$

$7^2 = 49$

$8^2 = 64$

we can tell that $\sqrt{50}$ must lie somewhere between 7 and 8. Also, it is much closer to 7 because 50 is just over 49.

So we can estimate that $\sqrt{50} \approx 7.1$.

(In fact, $7.1^2 = 50.41$)

EXERCISE 2.08 ANSWERS ON P. 547

Powers and roots U F R C

1 Write each expression using index notation. c

a $7 \times 7 \times 7$

b $4 \times 4 \times 4 \times 4 \times 4$

c $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$

d $-8 \times (-8) \times (-8) \times (-8) \times (-8) \times (-8) \times (-8)$

2 Evaluate each expression. EXAMPLE 23

a $(-8)^2$ **b** 2^3 **c** $(-6)^3$ **d** 3^4

e 7^1 **f** $(-2)^5$ **g** 10^3 **h** 4^4

i $(-11)^6$ **j** $3^9 - 9^3$ **k** 4×3^5 **l** $7^4 + 2 \times 8^2$

m $16^4 \div 2^{10} - 5^2$ **n** $(-4)^6 - 18 \div 3^2$

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3 Find the value of $\square$ to make each equation true. R

a $2^{\square} = 8$ **b** $3^{\square} = 27$ **c** $(-10)^{\square} = 100$

d $4^{\square} = 4096$ **e** $5^{\square} = 125$ **f** $(-3)^{\square} = -243$

4 What is the value of $(1.3)^2$? Select the correct answer **A**, **B**, **C** or **D**.

A 1.9 **B** 1.69 **C** 2.6 **D** 1.09

5 Which is smaller: 100^3 or 3^{100}?

6 Which is larger: 10^2 or 2^{10}?

7 **a** Evaluate $(2 \times 3)^2$ without using a calculator.

b Evaluate $2^2 \times 3^2$ without using a calculator.

c Does $(2 \times 3)^2 = 2^2 \times 3^2$? Explain your answer. C

8 **a** Evaluate $(4 \times 5)^2$.

b Evaluate $4^2 \times 5^2$.

c Does $(4 \times 5)^2 = 4^2 \times 5^2$? Explain your answer. C

9 Use the pattern you found in questions **7** and **8** to complete each equation. R C

a $(3 \times 8)^2 =$ _____ $\times$ _____ **b** $(a \times b)^2 =$ _____ $\times$ _____

10 Copy and complete each pattern. R C

a $18^2 = (6 \times 3)^2 = 6^2 \times$_____ = _____

b $22^2 = (2 \times 11)^2 =$ ____ $\times$ ____ = ____

c $16^2 = (2 \times$ ___$)^2 =$ ___ $\times$ ____ = _____

d $15^2 = ($___ $\times 5)^2 =$ ____ $\times$____ = ____

e $30^2 = (10 \times$ ___$)^2 =$ ___ $\times$ ____ = _____

f $28^2 = ($___ $\times$ ___$)^2 =$ ____ $\times$____ = ____

EXAMPLE 24

11 Evaluate each root.

a $\sqrt{784}$ **b** $\sqrt{256}$ **c** $\sqrt{289}$ **d** $\sqrt{1089}$

e $\sqrt[3]{8}$ **f** $\sqrt[3]{343}$ **g** $\sqrt[3]{2197}$ **h** $\sqrt[3]{216}$

12 Evaluate each root, correct to one decimal place.

a $\sqrt{37}$ **b** $\sqrt[3]{900}$ **c** $\sqrt{502}$ **d** $\sqrt[3]{6.5}$

e $\sqrt[3]{-495}$ **f** $\sqrt{2000}$ **g** $\sqrt{1.1}$ **h** $\sqrt[3]{1103}$

13 Copy and complete this table.

Number, x	1	2	3	4	5	6	7	8	9	10	11	12
Number squared, x^2				16			49					
Number cubed, x^3								512		1000		

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Use the table from question **13** to answer questions **14** to **17**.

EXAMPLE 25

14 Between which 2 consecutive whole numbers does $\sqrt{80}$ lie? Select **A**, **B**, **C** or **D**.

A 40 and 41 **B** 9 and 10 **C** 79 and 81 **D** 8 and 9

15 Between which 2 consecutive whole numbers does $\sqrt[3]{130}$ lie? Select **A**, **B**, **C** or **D**.

A 4 and 5 **B** 10 and 11 **C** 11 and 12 **D** 5 and 6

16 Between which 2 consecutive whole numbers does $\sqrt[3]{31}$ lie?

17 Estimate, correct to one decimal place, the value of each square root, then check your estimate using a calculator.

a $\sqrt{33}$ **b** $\sqrt{105}$ **c** $\sqrt{68}$

18 Select all the surds from this list of roots.

$\sqrt{6}$ $\sqrt{2025}$ $\sqrt{62}$ $\sqrt{961}$ $\sqrt{14}$ $\sqrt[3]{181}$ $\sqrt[3]{49}$ $\sqrt[3]{1000}$ $\sqrt[3]{674}$ $\sqrt[3]{1331}$

19 **a** Evaluate $\sqrt{4 \times 25}$.

b Evaluate $\sqrt{4} \times \sqrt{25}$.

c What do you notice about your answers to parts **a** and **b**? C

20 **a** Evaluate $\sqrt{27 \times 3}$.

b Evaluate $\sqrt{27} \times \sqrt{3}$.

c What do you notice about your answers to parts **a** and **b**? C

21 Use the pattern you found in questions **19** and **20** to complete each equation. R C

a $\sqrt{16 \times 9} =$ _____ $\times$ _____ **b** $\sqrt{a \times b} =$ _____ $\times$ _____

22 Copy and complete each pattern. R C

a $\sqrt{64} = \sqrt{4} \times \sqrt{16} =$ _____ $\times$ _____ $=$ _____

b $\sqrt{441} = \sqrt{9} \times \sqrt{} =$ _____ $\times$ _____ $=$ _____

c $\sqrt{144} = \sqrt{36} \times \sqrt{} =$ _____ $\times$ _____ $=$ _____

d $\sqrt{2025} = \sqrt{81} \times \sqrt{} =$ _____ $\times$ _____ $=$ _____

Technology

Powers and roots

1. Start a new spreadsheet and enter the label **Powers** in cell A1.
2. In cell A2, enter **=2^3** to evaluate 2^3. The caret symbol, ^ (shift 6), is the symbol for power on a spreadsheet.
3. In cell A3, enter **=3^5** to evaluate 3^5.
4. In cell A4, enter **=4^2** to evaluate 4^2.

5 Write an appropriate formula to evaluate each expression in the given cell.

A5: 6.1^2 A6: 4.25^2 A7: $5^2 \times 3^2$ A8: $(5 \times 3)^2$

6 Does your answer in cell A7 equal your answer in cell A8?

7 Write an appropriate formula to evaluate each expression in the given cell.

A9: 2.9^3 A10: 15^4 A11: 8.3^5 A12: $(-1)^3 + (-2)^4$ A13: $-1^3 + (-2^4)$

8 Does your answer in cell A12 equal your answer in cell A13?

9 In cell B1, enter **Square roots**.

10 In cell B2, enter **=sqrt(49)** to evaluate $\sqrt{49}$ (**=sqrt** stands for 'square root')

11 Write an appropriate formula to evaluate each square root in the given cell.

B3: $\sqrt{81}$ B4: $\sqrt{625}$ B5: $\sqrt{10000}$

B6: $\sqrt{71289}$ B7: $\sqrt{0.064}$ B8: $\sqrt{2.89}$

B9: $\sqrt{16.9}$ (Use **Format Cells** to round to 2 decimal places)

B10: $\sqrt{\frac{81}{25}}$

2.09 Factor trees

Factor trees

Factors and divisibility

Prime factors by repeated division

WS

Perfect and amicable numbers

Crossnumber puzzles

Crossnumber challenges

Every number can be written as a product of its prime factors. The prime factors can be found by using a **factor tree**.

Example 26

Write 60 as a product of its prime factors.

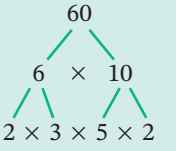
Solution

Draw a factor tree for 60.

Stop when all of the factors are prime.

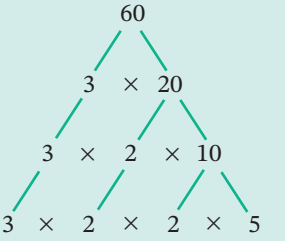
So 60 as a product of its prime factors is:

$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$



Note: It is possible to draw different factor trees using different factors for the same number, but the final list of prime factors should still be the same. Here is another factor tree for 60:

$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$



Using prime factors to find the highest common factor

Highest common factors

2.09

Highest common factor

The **highest common factor** (HCF) or **greatest common divisor** (GCD) of 2 (or more) numbers is the largest number that is a factor of **both (or all)** of these numbers.

To find the HCF of 2 numbers using their prime factors:

1 circle common prime factors

2 multiply them together.

Divisor is just another name for **factor**.

Example 27

Find the highest common factor of 36 and 45:

a by listing factors **b** using factor trees.

Solution

a Factors of 36 = **1**, 2, **3**, 4, 6, **9**, 12, 18, 36.

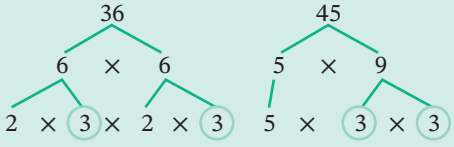
Factors of 45 = **1**, **3**, 5, **9**, 15, 30, 45.

The common factors are 1, 3, 9.

The highest common factor is **9**.

b
- Draw factor trees for 36 and 45
- Circle common prime factors: 3 and 3
- Multiply 3 and 3 to calculate the HCF

HCF = $3 \times 3 = \mathbf{9}$.



Using prime factors to find the lowest common multiple

Lowest common multiple

The **lowest common multiple** (LCM) of 2 (or more) numbers is the smallest number that is a multiple of **both (or all)** of these numbers.

To find the LCM of 2 numbers using their prime factors:

1 circle common prime factors

2 cross out **one** of each pair of common factors

3 multiply the remaining factors.

also called **least** common multiple

Example 28

Find the lowest common multiple of 36 and 45:

a by listing multiples **b** using factor trees.

Solution

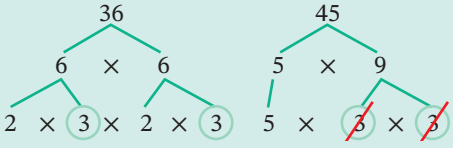
a Multiples of 36 = 36, 72, 108, 144, **180**, ...

Multiples of 45 = 45, 90, 105, **180**, 225, ...

The lowest common multiple is **180**.

b
- Draw factor trees for 36 and 45
- Circle common prime factors: 3 and 3
- Cross out the 2 3s in the second factor tree
- Multiply the remaining factors to calculate the LCM

LCM = $(2 \times 3 \times 2 \times 3) \times 5 = 36 \times 5 = \mathbf{180}$



Finding square and cube roots using prime factors

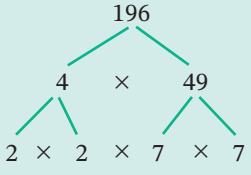
Example 29

Use a factor tree to find the value of:

a $\sqrt{196}$ **b** $\sqrt[3]{216}$

Solution

a Draw a factor tree for 196.



So $196 = 2 \times 2 \times 7 \times 7$

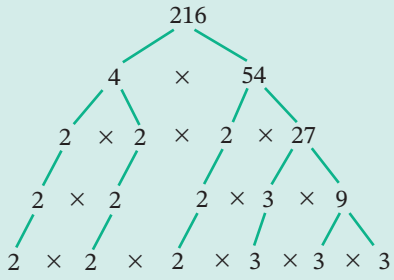
$\therefore \sqrt{196} = \sqrt{2 \times 2 \times 7 \times 7}$

$= 2 \times 7$

$= 14$

$\sqrt{2 \times 2} = 2, \ \sqrt{7 \times 7} = 7$

b Draw a factor tree for 216.



So $216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$

$\therefore \sqrt[3]{216} = \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3}$

$= 2 \times 3$

$= 6$

$\sqrt[3]{2 \times 2 \times 2} = 2, \sqrt[3]{3 \times 3 \times 3} = 3$

EXERCISE 2.09 ANSWERS ON P. 547

Factor trees UFRC

1 Use a factor tree to write each number as a product of its prime factors in index notation. R C

EXAMPLE 26

a 27 **b** 108 **c** 588 **d** 84

e 600 **f** 51 **g** 125 **h** 432

2 What is 42 expressed as a product of its prime factors? Select the correct answer **A**, **B**, **C** or **D**.

A 2×21 **B** $3 \times 3 \times 7$ **C** $2 \times 3 \times 7$ **D** 3×14

3 Draw 2 different factor trees for 80 and show that both give the same prime factors. R C

4 Find the highest common factor of each pair of numbers. R

EXAMPLE 27

a 8, 28 **b** 14, 35 **c** 20, 5 **d** 45, 60

e 24, 90 **f** 72, 24 **g** 16, 48 **h** 30, 18

5 Use factor trees to find the highest common factor of each pair of numbers. R C

a 45, 60 (check against answer to Question **4d**)

b 208, 78 **c** 33, 176

6 Find the lowest common multiple of each pair of numbers. R

EXAMPLE 28

a 9, 8 **b** 4, 6 **c** 10, 22 **d** 7, 4

e 6, 10 **f** 12, 8 **g** 9, 45 **h** 11, 10

7 Use factor trees to find the lowest common multiple of each pair of numbers. R C

a 90, 15 **b** 28, 36 **c** 16, 20

8 Use a factor tree to evaluate each root. R C

EXAMPLE 29

a $\sqrt{2025}$ **b** $\sqrt{441}$ **c** $\sqrt{256}$

d $\sqrt[3]{4096}$ **e** $\sqrt[3]{729}$ **f** $\sqrt[3]{5832}$

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9 Explain why the 'factor trees' method for finding the LCM works, using 28 and 36 from question **7b** as an example. R C

10 **a** Find the HCF of 28 and 76.

b Use the formula (rule)

$$\text{LCM of } a \text{ and } b = \frac{a \times b}{\text{HCF of } a \text{ and } b}.$$

to find the LCM of 28 and 76.

Investigation

Index laws for multiplying and dividing

The terms 3^6 and 3^2 have the same base, 3.

$3^6 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$ and $3^2 = 3 \times 3$.

What happens when we multiply them?

$3^6 \times 3^2 = (3 \times 3 \times 3 \times 3 \times 3 \times 3) \times (3 \times 3) = 3^8 = 6561$.

What happens when we divide 3^6 by 3^2?

$3^6 \div 3^2 = \frac{3 \times 3 \times 3 \times 3 \times \cancel{3^1} \times \cancel{3^1}}{\cancel{3^1} \times \cancel{3^1}} = 3 \times 3 \times 3 \times 3 = 3^4 = 81$.

1 Copy and complete each pattern.

a $2^4 \times 2^3 = (2 \times \_\_ \times \_\_ \times \_\_) \times (2 \times \_\_ \times \_\_) = 2^{\square} = \_\_\_\_$

b $7^2 \times 7^2 = (7 \times \_\_) \times (\_\_ \times \_\_) = 7^{\square} = \_\_\_\_$

c $10^2 \times 10^3 = (\_\_ \times \_\_) \times (\_\_ \times \_\_ \times \_\_) = 10^{\square} = \_\_\_\_$

d $4^3 \times 4^5 = (\_\_\_\_\_) \times (\_\_\_\_\_) = 4^{\square} = \_\_\_\_$

e $5^6 \times 5 = (\_\_\_\_\_) \times \_\_\_\_ = 5^{\square} = \_\_\_\_$

2 Copy and complete each pattern.

a $3^3 \times 3^7 = 3^{\square}$

b $2^5 \times 2^3 = 2^{\square}$

c $9^6 \times 9^4 = 9^{\square}$

d $6^3 \times 6^3 = 6^{\square}$

e $8^8 \times 8 = 8^{\square}$

3 Copy and complete each statement.

a $a^4 \times a^5 = \_\_\_\_\_$

b $a^m \times a^n = \_\_\_\_\_$

c When multiplying terms with the same base, _____ the powers.

4 Copy and complete each pattern.

a $2^5 \div 2^3 = \frac{2 \times 2 \times \cancel{2^1} \times \cancel{2^1} \times \cancel{2^1}}{\cancel{2^1} \times \cancel{2^1} \times \cancel{2^1}} = \_\_\_ \times \_\_\_ = 2^{\square} = \_\_\_\_$

b $7^8 \div 7^4 = \frac{7 \times 7 \times 7 \times \_\_ \times \_\_ \times \_\_ \times \_\_ \times \_\_}{7 \times 7 \times 7 \times \_\_} = \_\_\_ \times \_\_\_ \times \_\_\_ \times \_\_\_ = 7^{\square} = \_\_\_\_$

c $4^6 \div 4^3 = \frac{\_\_ \times \_\_ \times \_\_ \times \_\_ \times \_\_ \times \_\_}{\_\_ \times \_\_ \times \_\_} = \_\_\_ \times \_\_\_ \times \_\_\_ = 4^{\square} = \_\_\_\_$

d $3^6 \div 3 = \_\_\_\_\_ = \_\_\_\_\_ = 3^{\square} = \_\_\_\_$

e $10^7 \div 10^2 = \_\_\_\_\_ = \_\_\_\_\_ = 10^{\square} = \_\_\_\_$

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5 Copy and complete each pattern.

a $3^5 \div 3^2 = 3^{\square}$ **b** $10^9 \div 10^5 = 10^{\square}$

c $8^6 \div 8^4 = 8^{\square}$ **d** $6^7 \div 6 = 6^{\square}$

e $2^6 \div 2^5 = 2^{\square}$

6 Copy and complete each statement.

a $a^5 \div a^3 =$ _____ **b** $a^m \div a^n =$ _____

c When dividing terms with the same base, _____ the powers.

Index laws for multiplying and dividing 2.10

Index laws

When **multiplying terms with the same base**, *add* the powers.

$$a^m \times a^n = a^{m+n}$$

When **dividing terms with the same base**, *subtract* the powers.

$$a^m \div a^n = a^{m-n}$$

These are called **index laws** because **index** is another name for 'power'.

Example 30

Use index notation to simplify each product.

a $5^3 \times 5^4$ **b** $6^4 \times 6^5$ **c** $2 \times 2^3 \times 2^4$

Solution

a $5^3 \times 5^4 = 5^{3+4} = 5^7$ Adding the powers

b $6^4 \times 6^5 = 6^{4+5} = 6^9$ Note: Do **not** multiply the bases

c $2 \times 2^3 \times 2^4 = 2^{1+3+4} = 2^8$ Note: $2 = 2^1$

Example 31

Use index notation to simplify each quotient.

a $5^8 \div 5^3$ **b** $\frac{7^5}{7^2}$ **c** $6^{11} \div 6^{10}$

Solution

a $5^8 \div 5^3 = 5^{8-3} = 5^5$ Subtracting the powers

b $7^5 \div 7^2 = 7^{5-2} = 7^3$ Note: Do **not** divide the bases

c $6^{11} \div 6^{10} = 6^{11-10} = 6^1 = 6$

EXERCISE 2.10 ANSWERS ON P. 548

Index laws for multiplying and dividing U F R C

EXAMPLE 30

1 Use index notation to simplify each product. R C

a $6^3 \times 6^2$ **b** $4^3 \times 4^4$ **c** $2^5 \times 2^6$ **d** 8×8^9

e $11^4 \times 11^4$ **f** $7^3 \times 7^7$ **g** 5×5^6 **h** $6^6 \times 6^{10}$

i $2^3 \times 2^2 \times 2^4$ **j** $5^3 \times 5^4 \times 5^2$ **k** $2^4 \times 2^4 \times 2^6$ **l** $3^2 \times 3^9 \times 3^3 \times 3^2$

2 Which term is equal to $2^3 \times 2^3 \times 2$? Select the correct answer **A**, **B**, **C** or **D**.

A 2^6 **B** 2^9 **C** 8^7 **D** 2^7

3 Simplify each product, then evaluate it as a whole number.

a $5^2 \times 5^4$ **b** $7^6 \times 7^2$ **c** $3^4 \times 3^9$ **d** $4^3 \times 4^3$

e $9^2 \times 9^5$ **f** $6^5 \times 6^4$ **g** $8^4 \times 8^6$ **h** 7×7^3

EXAMPLE 31

4 Use index notation to simplify each quotient. C

a $2^6 \div 2^2$ **b** $3^9 \div 3^7$ **c** $7^8 \div 7^2$ **d** $\frac{2^9}{2^4}$

e $5^{11} \div 5^9$ **f** $10^{11} \div 10^7$ **g** $11^{12} \div 11^5$ **h** $6^{18} \div 6^{11}$

i $2^5 \div 2^4$ **j** $\frac{16^{13}}{16^7}$ **k** $35^{12} \div 35$ **l** $\frac{20^7}{20^6}$

5 Which term is equal to $5^8 \div 5^2$? Select **A**, **B**, **C** or **D**.

A 1^6 **B** 5^4 **C** 1^5 **D** 5^6

6 Simplify each quotient, then evaluate it as a whole number.

a $2^6 \div 2^2$ **b** $5^9 \div 5^7$ **c** $4^5 \div 4^2$ **d** $10^9 \div 10^4$

e $\frac{8^{11}}{8^8}$ **f** $6^{10} \div 6^2$ **g** $11^4 \div 11^3$ **h** $\frac{3^7}{3}$

7 Copy and complete this table of powers.

Number, n	n^2	n^3	n^4	n^5	n^6
2	4	8			
3		27		243	
4	16		256		
5		125			15 625

8 Use your answers from question **7** to evaluate each expression.

a $2^3 \times 2^2$ **b** $3^2 \times 3^2$ **c** $5^2 \times 5$ **d** $5^3 \times 5^2$

e $4^4 \times 4^2$ **f** $2^2 \times 3^2 \times 3^3 \times 2$ **g** $4^5 \div 4^2$ **h** $5^6 \div 5$

i $\frac{3^6}{3^4}$ **j** $2^5 \div 2^4$ **k** $\frac{4^4}{4^3}$ **l** $3^3 \div 3^3$

9 Copy and complete each equation. R

a $5^2 \times$ ____ $= 5^6$ **b** ____ $\div 3^4 = 3^3$ **c** $2^7 \div$ ____ $= 2^4$

10 Simplify each algebraic expression. R C

a $a^3 \times a^7$ **b** $x^2 \times x^4$ **c** $b^4 \times b$ **d** $y^3 \times y^2$

e $d^6 \div d^2$ **f** $x^5 \div x^3$ **g** $\frac{a^8}{a^7}$ **h** $\frac{r^5}{r}$

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Investigation

More index laws

What happens when we raise 2^3 to a power of 4? How do we simplify $(2^3)^4$ using index notation?

$(2^3)^4 = 2^3 \times 2^3 \times 2^3 \times 2^3 = 2^{3+3+3+3} = 2^{12}$

1 Copy and complete each pattern.

a $(3^2)^5 = \_\_ \times \_\_ \times \_\_ \times \_\_ \times \_\_ = 3$ —— $= 3$–

b $(7^4)^2 = \_\_ \times \_\_ = 7$ — $= 7$–

c $(16^3)^3 = \_\_\_\_ \times \_\_\_\_ \times \_\_\_\_ = 16$ —— $= 16$–

d $(2^6)^4 = \_\_ \times \_\_ \times \_\_ \times \_\_ = 2$ —— $= 2$–

e $(5^2)^3 = \_\_ \times \_\_ \times \_\_ = 5$—— $= 5$–

2 Copy and complete each statement.

a $(a^5)^2 =$ _____

b $(a^m)^n =$ _____

c When a term with a power is raised to another power, _____ the powers.

3 We know that $3^2 = 3 \times 3$ and $3^1 = 3$, but what is the value of 3^0 (3 to the power of 0)?

a Copy and complete this pattern.

$3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$

$3^4 = 3 \times 3 \times 3 \times 3 = 81$

$3^3 = 3 \times \_\_\_ \times \_\_\_ = \_\_\_$

$3^2 =$ _____ $= \_\_\_$

$3^1 =$ _____ $= \_\_\_$

b As the powers of 3 decrease by 1, what happens to its calculated value?

c According to the pattern, what should be the value of 3^0?

4 **a** Use index notation to simplify $2^5 \div 2^5$.

b But $2^5 \div 2^5 = \frac{2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2}$. What is the answer to this?

c What is any number divided by itself equal to?

d What is the value of 2^0?

5 **a** Use index notation to simplify $5^2 \times 5^0$. What do you notice about the answer?

b If a number multiplied by 5^0 equals itself, what must be the value of 5^0?

c Use index notation to simplify $10^6 \div 10^0$. What do you notice about the answer?

d If a number divided by 10^0 equals itself, what must be the value of 10^0?

6 Copy and complete each statement.

a $4^0 =$ _____

b $a^0 =$ _____

c When any number is raised to a power of zero, the answer is always _____.

2.11 More index laws

WS What is the power question?

WS Power calculations

More index laws

When a **term with a power** is raised to **another power**, **multiply** the powers.

$$(a^m)^n = a^{m \times n}$$

Any number raised to the **power of zero** is equal to **1**.

$$a^0 = 1$$

Index laws

Example 32

Use index notation to simplify each expression.

a $(6^2)^5$ **b** $(7^3)^3$

Solution

a $(6^2)^5 = 6^{2 \times 5} = 6^{10}$ Multiply the powers

b $(7^3)^3 = 7^{3 \times 3} = 7^9$

Index laws

Example 33

Evaluate each expression.

a 6^0 **b** $(3 \times 4)^0$ **c** 3×4^0

Solution

a $6^0 = 1$ Any number raised to the power of 0 equals 1.

b $(3 \times 4)^0 = 12^0 = 1$

c $3 \times 4^0 = 3 \times 1 = 3$

EXERCISE 2.11 ANSWERS ON P. 548

More index laws U F R C

EXAMPLE 32

1 Use index notation to simplify each expression. R C

a $(2^9)^2$ **b** $(5^2)^3$ **c** $(7^4)^5$ **d** $(8^2)^7$

e $(3^8)^3$ **f** $(9^6)^4$ **g** $(6^7)^2$ **h** $(11^5)^5$

i $(8^{10})^3$ **j** $(4^9)^5$ **k** $[(-2)^4]^7$ **l** $[(-7)^3]^6$

2 Which term is equal to $(3^4)^2$? Select the correct answer **A**, **B**, **C** or **D**.

A 9^6 **B** 6^4 **C** 3^6 **D** 3^8

3 Simplify each expression, then evaluate it as a whole number.

a $(5^2)^4$ **b** $(2^5)^3$ **c** $(3^9)^2$ **d** $(6^3)^4$

e $(7^2)^3$ **f** $(4^6)^2$ **g** $(8^0)^5$ **h** $(4^3)^3$

EXAMPLE 33

4 Evaluate each expression.

a 9^0 **b** 3^0 **c** 10^0 **d** 16^0

e $(-2)^0$ **f** 6^0 **g** $(7-3)^0$ **h** $7-3^0$

i $(-5)^0$ **j** 7×3^0 **k** $(7\times3)^0$ **l** $2+3^0$

m $(6+6)^0$ **n** 6^0+6^0 **o** $6\div6^0$ **p** $10-10^0$

5 Evaluate $6^2\div2^0$. Select **A**, **B**, **C** or **D**.

A 3 **B** 36 **C** 18 **D** 6

6 Copy and complete this table of powers of 2.

2^0	2^1	2^2	2^3	2^4	2^5	2^6	2^7	2^8	2^9	2^{10}	2^{11}	2^{12}
		4			32					1024		

7 Use your answers from question **6** to evaluate each expression. R

a $(2^2)^3$ **b** $(2^6)^2$ **c** $(2^2)^2$ **d** $(2^3)^3$

e $(2^0)^4$ **f** $(2^2)^4$ **g** $(2^{10})^0$ **h** $(2^7)^1$

8 Copy and complete each equation. R C

a $(6^2)^- = 6^8$ **b** $(5^-)^3 = 5^9$ **c** $(10^-)^- = 10^{18}$

9 Simplify each algebraic expression. R C

a $(a^4)^2$ **b** $(x^6)^6$ **c** $(d^2)^5$ **d** $(n^3)^6$

e p^0 **f** $4\times y^0$ **g** $(4\times y)^0$ **h** a^0-b^0

2.11

Foundation Standard Complex

Crossnumber challenges

Power plus ANSWERS ON P. 548

1 Even if we don't know the exact value of a surd, we can use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ to simplify a surd if one of its factors is a square number. Study each example:

- $\sqrt{40} = \sqrt{4 \times 10} = \sqrt{4} \times \sqrt{10} = 2\sqrt{10}$
- $\sqrt{32} = \sqrt{16 \times 2} = \sqrt{16} \times \sqrt{2} = 4\sqrt{2}$
- $\sqrt{36} = \sqrt{9 \times 4} = \sqrt{9} \times \sqrt{4} = 3 \times 2 = 6$
- $\sqrt{125} = \sqrt{25 \times 5} = \sqrt{25} \times \sqrt{5} = 5\sqrt{5}$

Simplify each surd.

a	$\sqrt{81}$	**b**	$\sqrt{8}$	**c**	$\sqrt{27}$	**d**	$\sqrt{50}$
e	$\sqrt{72}$	**f**	$\sqrt{45}$	**g**	$\sqrt{180}$	**h**	$\sqrt{128}$
i	$\sqrt{28}$	**j**	$\sqrt{200}$	**k**	$\sqrt{48}$	**l**	$\sqrt{245}$

2 **a** Use index notation to simplify $2^4 \div 2^5$.

b But $2^4 \div 2^5 = \dfrac{2 \times 2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2}$. What is the answer to this?

c What is the value of 2^{-1}?

d What is the value of 2^{-2}?

3 **a** Does $(-2)^5 = -2^5$?

b Does $(-3)^4 = -3^4$?

c For what values of n does $(-1)^n = -1$?

d For what value of x is x^0 not defined?

4 Scientific notation is a special way of writing very large or very small numbers using powers of 10. For example:

- $25\,600$ can be written as $2.56 \times 10 \times 10 \times 10 \times 10 = 2.56 \times 10^4$ because $2.56 \times 10^4 = 2.56 \times 10 \times 10 \times 10 \times 10$
- 0.001678 can be written as $1.678 \div 10 \div 10 \div 10 = 1.678 \times 10^{-3}$ because $1.678 \times 10^{-3} = 1.678 \div 10 \div 10 \div 10$

Note that the first number is a decimal between 1 and 10 and the second number is a power of 10.

Convert each number written in scientific notation to an ordinary number.

a	2.4×10^4	**b**	4.55×10^5	**c**	9.33×10^{-2}
d	6.38×10^{-2}	**e**	8.7×10^6	**f**	5.82×10^{-3}
g	1.26×10^{-1}	**h**	2.69×10^3	**i**	3.14×10^8

Can you see a quick way of writing the answer each time?

5 Write each number in scientific notation.

a	12 000	**b**	345 000 000	**c**	0.007
d	4000	**e**	0.0005	**f**	0.00041
g	1 920 000	**h**	0.000 361	**i**	0.000 000 063 7

6 **a** What is the largest number that can be displayed on your calculator?

b What is the smallest?

CHAPTER 2 REVIEW

Qz Whole numbers

Language of maths

base	consecutive	cube root	decimal places
estimate	evaluate	factor tree	highest common factor
grouping symbol	index laws	index notation	integer
long division	lowest common multiple	mental calculation	order of operations
prime factors	product	quotient	recurring decimal
round	square root	surd	terminating decimal

1 If you round a decimal to the nearest hundredth, how many decimal places is this?

2 Write a numerical expression that requires the use of:

a 'order of operations' rules **b** long division

3 Write 'the cube root of −64' using mathematical symbols, then evaluate it.

4 In 8^5, what are the names given to the '8' and the '5'?

5 What is another name for:

a grouping symbols? **b** index?

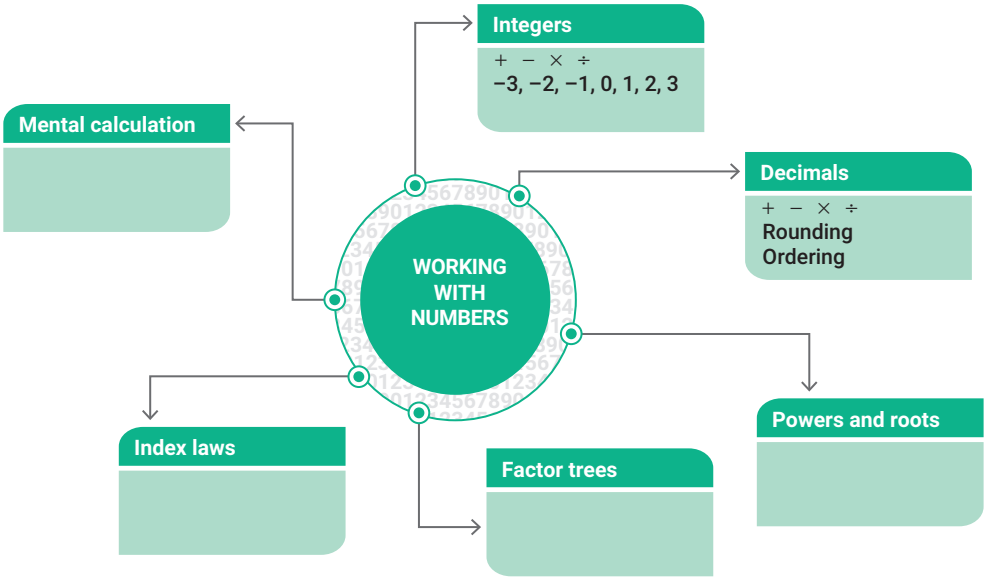
6 What diagram is used to write a number as a product of its prime factors?

Topic summary

- What parts of this chapter do you remember from last year?
- Are there any parts of this chapter that you do not understand?
- Discuss any problems with your teacher or a friend.

WS Mind map: Working with numbers

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



TEST YOURSELF 2

ANSWERS ON P. 549

2.01 **1** Use mental calculation strategies to evaluate each expression.

a 7×200 **b** 14×15 **c** $8000 \div 50$

d $\frac{1}{2} \times 262$ **e** $68 + 19$ **f** 35×11

g 65×4 **h** 8×60 **i** $26 + 61 + 156 + 19 + 14$

j $8 \times 25 \times 4$ **k** 23×12 **l** $168 \div 4$

m $2700 \div 300$ **n** 40×70 **o** 37×9

p 18×5 **q** $384 \div 8$ **r** $970 - 245$

s $4 \times 8 \times 5$

2.01 **2** Evaluate each quotient.

a $812 \div 7$ **b** $846 \div 9$ **c** $396 \div 18$ **d** $2139 \div 23$

2.02 **3** Evaluate each expression.

a $-4 + 6$ **b** $5 - (-2)$ **c** $-3 - 8$ **d** $-2 - (-2)$

2.02 **4** Death Valley in the USA is 397 m below sea level and Mt Everest is 8840 m above sea level. What is the vertical distance between the bottom of Death Valley and the top of Mt Everest?

2.02 **5** On 17 July, the maximum temperature in Armidale was 17°C and the minimum temperature was − 6°C. By how much had the temperature risen during the day?

2.03 **6** Evaluate each product.

a -2×7 **b** $-3 \times (-8)$ **c** $(-7)^2$ **d** $16 \times (-5)$

2.03 **7** Evaluate each quotient.

a $36 \div (-4)$ **b** $-28 \div 7$ **c** $24 \div (-2)$ **d** $-15 \div (-5)$

2.04 **8** Evaluate each expression.

a $2 + (13 - 8) \times 11$ **b** $14 - 5 \times (-3)$ **c** $15 - 18 \div 3 \times 2$

d $80 - [(7 - 10) \times (-6)]$ **e** $\frac{8-3}{100 \div 4}$ **f** $\frac{15 \times (-6+8)}{24 \div 2^2}$

2.05 **9** Write these decimals in descending order: 0.417, 0.47, 0.147, 0.471.

2.05 **10** Write each number correct to the number of decimal places shown in the brackets.

a 0.473 [1] **b** 13.1051 [2] **c** 98.4273 [3]

2.05 **11** To complete the wiring of a house, Joe the electrician needed these lengths of cables: 2.3 m, 1.9 m, 4.2 m and 3.8 m. How much cable did he need altogether?

2.06 **12** Evaluate each expression.

a $37.4 - 6.9$ **b** $13.3 + 0.82 + 5.6$ **c** 2.6×4 **d** 5.5×0.6

e 0.71×1.3 **f** $(2.5)^2$ **g** $9.57 \div 4$ **h** $8.12 \div 0.7$

2.06 **13** A drink bottle holds 0.5 L. How many drink bottles can be filled from a container that holds 7.5 L?

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14 Convert each fraction to a decimal. 2.07

a $\frac{2}{9}$ **b** $\frac{7}{8}$ **c** $\frac{1}{6}$

15 Evaluate each expression. 2.08

a 7^3 **b** 11^5 **c** $(-5)^4$

d $\sqrt{400}$ **e** $\sqrt[3]{27}$ **f** $\sqrt[3]{-125}$

16 Without using a calculator, estimate $\sqrt{31}$, correct to one decimal place. 2.08

17 Copy and complete: 2.08

a $20^2 = (\_\_ \times 10)^2$

$= \_\_^2 \times 10^2$

$= \_\_ \times \_\_$

$= \_\_$

b $\sqrt{196} = \sqrt{\ } \times \sqrt{49}$

$= \_\_ \times \_\_$

$= \_\_$

18 **a** Find the highest common factor of 36 and 42. 2.09

b Find the lowest common multiple of 20 and 14.

19 Use factor trees to write 132 and 88 as products of their prime factors, then use them to find the highest common factor of 132 and 88. 2.09

20 Use factor trees to write 12 and 45 as products of their prime factors, then use them to find the lowest common multiple of 12 and 45. 2.09

21 Use a factor tree to write 784 as a product of its prime factors, then use it to evaluate $\sqrt{784}$. 2.09

22 Simplify each expression using index notation. 2.10

a $4^2 \times 4^5$ **b** $6^4 \times 6^7$ **c** $12^4 \times 12^4$ **d** $9^3 \times 9$

e $2^7 \div 2^3$ **f** $5^9 \div 5^6$ **g** $3^8 \div 3^7$ **h** $6^{11} \div 6^4 \times 6^5$

23 Simplify each expression using index notation. 2.11

a $(3^4)^2$ **b** $(7^2)^5$ **c** $(6^3)^3$ **d** $(3^5)^3$

24 Evaluate each expression. 2.11

a 3^0 **b** 6^0 **c** 4×4^0 **d** $9 - 9^0$

e 10^0 **f** $(12 \times 3)^0$ **g** $(5 - 3)^0$ **h** $5^0 - 3^0$

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